```
(*Define the polynomial g*)
```

In[14]:=
```
g[γ_, k_] := 4 (-1 + k)^2 (-k (2 + k) (-1 + γ^(1+k))^2 + (1 + k)^2 (-1 + γ^k) (-1 + γ^(2+k)))^3 -
  (2 k^2 (2 + k) (3 + k) (-1 + γ^(1+k))^3 -
    3 k (1 + k)^2 (3 + k) (-1 + γ^k) (-1 + γ^(1+k)) (-1 + γ^(2+k)) + (1 + k)^3 (2 + k) (-1 + γ^k)^2 (-1 + γ^(3+k)))^2
```

```
(*Here is the form shown in the paper*)
```

In[26]:=
```
TraditionalForm[g[γ, k]]
```

Out[26]//TraditionalForm=

$$4 (k-1)^2 \left((k+1)^2 \left(\gamma^k-1\right)\left(\gamma^{k+2}-1\right)-k (k+2) \left(\gamma^{k+1}-1\right)^2\right)^3 - \left(2 k^2 (k+2) (k+3) \left(\gamma^{k+1}-1\right)^3+(k+2) (k+1)^3 \left(\gamma^k-1\right)^2 \left(\gamma^{k+3}-1\right)-3 k (k+3) (k+1)^2 \left(\gamma^k-1\right)\left(\gamma^{k+1}-1\right)\left(\gamma^{k+2}-1\right)\right)^2$$

```
(*We first assume that k≥5. The case of 1≤k≤4 will be handled later.*)

(*Factorize g*)
```

In[27]:=
```
Factor[g[γ, k]]
```

Out[27]=
```
-(1 + k)^3 (-1 + γ)^2 γ^k
 (-4 + 8 k - 4 k^2 - 8 γ + 4 k γ + 4 k^2 γ + 12 γ^k - 15 k^2 γ^k - 5 k^3 γ^k + 7 k^4 γ^k + k^5 γ^k - 12 γ^(2 k) - 24 k γ^(2 k) -
   6 k^2 γ^(2 k) + 18 k^3 γ^(2 k) + 18 k^4 γ^(2 k) + 6 k^5 γ^(2 k) + 4 γ^(3 k) + 16 k γ^(3 k) + 25 k^2 γ^(3 k) + 19 k^3 γ^(3 k) + 7 k^4 γ^(3 k) +
   k^5 γ^(3 k) + 24 γ^(1+k) - 12 k γ^(1+k) + 36 k^2 γ^(1+k) - 16 k^3 γ^(1+k) - 28 k^4 γ^(1+k) - 4 k^5 γ^(1+k) - 48 k γ^(2+k) + 6 k^2 γ^(2+k) +
   66 k^3 γ^(2+k) + 42 k^4 γ^(2+k) + 6 k^5 γ^(2+k) + 8 γ^(3+k) - 4 k γ^(3+k) - 52 k^2 γ^(3+k) - 64 k^3 γ^(3+k) - 28 k^4 γ^(3+k) - 4 k^5 γ^(3+k) +
   4 γ^(4+k) + 16 k γ^(4+k) + 25 k^2 γ^(4+k) + 19 k^3 γ^(4+k) + 7 k^4 γ^(4+k) + k^5 γ^(4+k) - 24 γ^(1+2 k) + 12 k γ^(1+2 k) + 12 k^2 γ^(1+2 k) -
   48 k^3 γ^(1+2 k) - 72 k^4 γ^(1+2 k) - 24 k^5 γ^(1+2 k) + 96 k γ^(2+2 k) - 12 k^2 γ^(2+2 k) + 60 k^3 γ^(2+2 k) + 108 k^4 γ^(2+2 k) +
   36 k^5 γ^(2+2 k) - 24 γ^(3+2 k) + 12 k γ^(3+2 k) + 12 k^2 γ^(3+2 k) - 48 k^3 γ^(3+2 k) - 72 k^4 γ^(3+2 k) - 24 k^5 γ^(3+2 k) - 12 γ^(4+2 k) -
   24 k γ^(4+2 k) - 6 k^2 γ^(4+2 k) + 18 k^3 γ^(4+2 k) + 18 k^4 γ^(4+2 k) + 6 k^5 γ^(4+2 k) + 8 γ^(1+3 k) - 4 k γ^(1+3 k) - 52 k^2 γ^(1+3 k) -
   64 k^3 γ^(1+3 k) - 28 k^4 γ^(1+3 k) - 4 k^5 γ^(1+3 k) - 48 k γ^(2+3 k) + 6 k^2 γ^(2+3 k) + 66 k^3 γ^(2+3 k) + 42 k^4 γ^(2+3 k) + 6 k^5 γ^(2+3 k) +
   24 γ^(3+3 k) - 12 k γ^(3+3 k) + 36 k^2 γ^(3+3 k) - 16 k^3 γ^(3+3 k) - 28 k^4 γ^(3+3 k) - 4 k^5 γ^(3+3 k) + 12 γ^(4+3 k) - 15 k^2 γ^(4+3 k) -
   5 k^3 γ^(4+3 k) + 7 k^4 γ^(4+3 k) + k^5 γ^(4+3 k) - 8 γ^(3+4 k) + 4 k γ^(3+4 k) + 4 k^2 γ^(3+4 k) - 4 γ^(4+4 k) + 8 k γ^(4+4 k) - 4 k^2 γ^(4+4 k))
```

```
(*Extract the non-negative factors*)
```

In[28]:=
```
g0[γ_, k_] = Factor[g[γ, k]] / ((1 + k)^3 (-1 + γ)^2 γ^k)
```

Out[28]=
```
4 - 8 k + 4 k^2 + 8 γ - 4 k γ - 4 k^2 γ - 12 γ^k + 15 k^2 γ^k + 5 k^3 γ^k - 7 k^4 γ^k - k^5 γ^k + 12 γ^(2 k) + 24 k γ^(2 k) +
 6 k^2 γ^(2 k) - 18 k^3 γ^(2 k) - 18 k^4 γ^(2 k) - 6 k^5 γ^(2 k) - 4 γ^(3 k) - 16 k γ^(3 k) - 25 k^2 γ^(3 k) - 19 k^3 γ^(3 k) - 7 k^4 γ^(3 k) -
 k^5 γ^(3 k) - 24 γ^(1+k) + 12 k γ^(1+k) - 36 k^2 γ^(1+k) + 16 k^3 γ^(1+k) + 28 k^4 γ^(1+k) + 4 k^5 γ^(1+k) + 48 k γ^(2+k) - 6 k^2 γ^(2+k) -
 66 k^3 γ^(2+k) - 42 k^4 γ^(2+k) - 6 k^5 γ^(2+k) - 8 γ^(3+k) + 4 k γ^(3+k) + 52 k^2 γ^(3+k) + 64 k^3 γ^(3+k) + 28 k^4 γ^(3+k) + 4 k^5 γ^(3+k) -
 4 γ^(4+k) - 16 k γ^(4+k) - 25 k^2 γ^(4+k) - 19 k^3 γ^(4+k) - 7 k^4 γ^(4+k) - k^5 γ^(4+k) + 24 γ^(1+2 k) - 12 k γ^(1+2 k) - 12 k^2 γ^(1+2 k) +
 48 k^3 γ^(1+2 k) + 72 k^4 γ^(1+2 k) + 24 k^5 γ^(1+2 k) - 96 k γ^(2+2 k) + 12 k^2 γ^(2+2 k) - 60 k^3 γ^(2+2 k) - 108 k^4 γ^(2+2 k) -
 36 k^5 γ^(2+2 k) + 24 γ^(3+2 k) - 12 k γ^(3+2 k) - 12 k^2 γ^(3+2 k) + 48 k^3 γ^(3+2 k) + 72 k^4 γ^(3+2 k) + 24 k^5 γ^(3+2 k) + 12 γ^(4+2 k) +
 24 k γ^(4+2 k) + 6 k^2 γ^(4+2 k) - 18 k^3 γ^(4+2 k) - 18 k^4 γ^(4+2 k) - 6 k^5 γ^(4+2 k) - 8 γ^(1+3 k) + 4 k γ^(1+3 k) + 52 k^2 γ^(1+3 k) +
 64 k^3 γ^(1+3 k) + 28 k^4 γ^(1+3 k) + 4 k^5 γ^(1+3 k) + 48 k γ^(2+3 k) - 6 k^2 γ^(2+3 k) - 66 k^3 γ^(2+3 k) - 42 k^4 γ^(2+3 k) - 6 k^5 γ^(2+3 k) -
 24 γ^(3+3 k) + 12 k γ^(3+3 k) - 36 k^2 γ^(3+3 k) + 16 k^3 γ^(3+3 k) + 28 k^4 γ^(3+3 k) + 4 k^5 γ^(3+3 k) - 12 γ^(4+3 k) + 15 k^2 γ^(4+3 k) +
 5 k^3 γ^(4+3 k) - 7 k^4 γ^(4+3 k) - k^5 γ^(4+3 k) + 8 γ^(3+4 k) - 4 k γ^(3+4 k) - 4 k^2 γ^(3+4 k) + 4 γ^(4+4 k) - 8 k γ^(4+4 k) + 4 k^2 γ^(4+4 k)
```

```
(*The non-negativity of g and g0 for γ≥1 is equivalent.*)

(*Check g0[1,k]*)
```

In[29]:=
```
g0[1, k]
```

Out[29]=
```
0
```

```
(*Since g0[1,k] = 0 for any k,
it suffices to show that the derivative of g0 is non-negative for γ≥1*)
```

In[30]:=
```
g1[γ_, k_] = D[g0[γ, k], γ]
```

Out[30]=
```
8 - 4 k - 4 k^2 - 12 k γ^(-1+k) + 15 k^3 γ^(-1+k) + 5 k^4 γ^(-1+k) - 7 k^5 γ^(-1+k) - k^6 γ^(-1+k) - 24 (1 + k) γ^k +
 12 k (1 + k) γ^k - 36 k^2 (1 + k) γ^k + 16 k^3 (1 + k) γ^k + 28 k^4 (1 + k) γ^k + 4 k^5 (1 + k) γ^k +
 24 (1 + 2 k) γ^(2 k) - 12 k (1 + 2 k) γ^(2 k) - 12 k^2 (1 + 2 k) γ^(2 k) + 48 k^3 (1 + 2 k) γ^(2 k) + 72 k^4 (1 + 2 k) γ^(2 k) +
 24 k^5 (1 + 2 k) γ^(2 k) - 8 (1 + 3 k) γ^(3 k) + 4 k (1 + 3 k) γ^(3 k) + 52 k^2 (1 + 3 k) γ^(3 k) + 64 k^3 (1 + 3 k) γ^(3 k) +
 28 k^4 (1 + 3 k) γ^(3 k) + 4 k^5 (1 + 3 k) γ^(3 k) + 48 k (2 + k) γ^(1+k) - 6 k^2 (2 + k) γ^(1+k) - 66 k^3 (2 + k) γ^(1+k) -
 42 k^4 (2 + k) γ^(1+k) - 6 k^5 (2 + k) γ^(1+k) - 8 (3 + k) γ^(2+k) + 4 k (3 + k) γ^(2+k) + 52 k^2 (3 + k) γ^(2+k) +
 64 k^3 (3 + k) γ^(2+k) + 28 k^4 (3 + k) γ^(2+k) + 4 k^5 (3 + k) γ^(2+k) - 4 (4 + k) γ^(3+k) - 16 k (4 + k) γ^(3+k) -
 25 k^2 (4 + k) γ^(3+k) - 19 k^3 (4 + k) γ^(3+k) - 7 k^4 (4 + k) γ^(3+k) - k^5 (4 + k) γ^(3+k) + 24 k γ^(-1+2 k) +
 48 k^2 γ^(-1+2 k) + 12 k^3 γ^(-1+2 k) - 36 k^4 γ^(-1+2 k) - 36 k^5 γ^(-1+2 k) - 12 k^6 γ^(-1+2 k) - 96 k (2 + 2 k) γ^(1+2 k) +
 12 k^2 (2 + 2 k) γ^(1+2 k) - 60 k^3 (2 + 2 k) γ^(1+2 k) - 108 k^4 (2 + 2 k) γ^(1+2 k) - 36 k^5 (2 + 2 k) γ^(1+2 k) +
 24 (3 + 2 k) γ^(2+2 k) - 12 k (3 + 2 k) γ^(2+2 k) - 12 k^2 (3 + 2 k) γ^(2+2 k) + 48 k^3 (3 + 2 k) γ^(2+2 k) +
 72 k^4 (3 + 2 k) γ^(2+2 k) + 24 k^5 (3 + 2 k) γ^(2+2 k) + 12 (4 + 2 k) γ^(3+2 k) + 24 k (4 + 2 k) γ^(3+2 k) +
 6 k^2 (4 + 2 k) γ^(3+2 k) - 18 k^3 (4 + 2 k) γ^(3+2 k) - 18 k^4 (4 + 2 k) γ^(3+2 k) - 6 k^5 (4 + 2 k) γ^(3+2 k) - 12 k γ^(-1+3 k) -
 48 k^2 γ^(-1+3 k) - 75 k^3 γ^(-1+3 k) - 57 k^4 γ^(-1+3 k) - 21 k^5 γ^(-1+3 k) - 3 k^6 γ^(-1+3 k) + 48 k (2 + 3 k) γ^(1+3 k) -
 6 k^2 (2 + 3 k) γ^(1+3 k) - 66 k^3 (2 + 3 k) γ^(1+3 k) - 42 k^4 (2 + 3 k) γ^(1+3 k) - 6 k^5 (2 + 3 k) γ^(1+3 k) -
 24 (3 + 3 k) γ^(2+3 k) + 12 k (3 + 3 k) γ^(2+3 k) - 36 k^2 (3 + 3 k) γ^(2+3 k) + 16 k^3 (3 + 3 k) γ^(2+3 k) +
 28 k^4 (3 + 3 k) γ^(2+3 k) + 4 k^5 (3 + 3 k) γ^(2+3 k) - 12 (4 + 3 k) γ^(3+3 k) + 15 k^2 (4 + 3 k) γ^(3+3 k) +
 5 k^3 (4 + 3 k) γ^(3+3 k) - 7 k^4 (4 + 3 k) γ^(3+3 k) - k^5 (4 + 3 k) γ^(3+3 k) + 8 (3 + 4 k) γ^(2+4 k) -
 4 k (3 + 4 k) γ^(2+4 k) - 4 k^2 (3 + 4 k) γ^(2+4 k) + 4 (4 + 4 k) γ^(3+4 k) - 8 k (4 + 4 k) γ^(3+4 k) + 4 k^2 (4 + 4 k) γ^(3+4 k)
```

```
(*Check g1[1,k]*)
```

In[32]:=
```
Simplify[g1[1, k]]
```

Out[32]=
```
0
```

```
(*Since g1[1,k] = 0 for any k,
it suffices to show that the derivative of g1 is non-negative for γ≥1*)

(*Factorize the derivative of g1*)
```

In[34]:=
```
Factor[D[g1[γ, k], γ]]
```

Out[34]=
$$\gamma^{-2+k} \left(12\,k - 12\,k^2 - 15\,k^3 + 10\,k^4 + 12\,k^5 - 6\,k^6 - k^7 - 24\,k\,\gamma - 12\,k^2\,\gamma - 24\,k^3\,\gamma - 20\,k^4\,\gamma + 44\,k^5\,\gamma + 32\,k^6\,\gamma + 4\,k^7\,\gamma + 96\,k\,\gamma^2 + 132\,k^2\,\gamma^2 - 102\,k^3\,\gamma^2 - 288\,k^4\,\gamma^2 - 204\,k^5\,\gamma^2 - 60\,k^6\,\gamma^2 - 6\,k^7\,\gamma^2 - 48\,\gamma^3 - 16\,k\,\gamma^3 + 324\,k^2\,\gamma^3 + 648\,k^3\,\gamma^3 + 540\,k^4\,\gamma^3 + 228\,k^5\,\gamma^3 + 48\,k^6\,\gamma^3 + 4\,k^7\,\gamma^3 - 48\,\gamma^4 - 220\,k\,\gamma^4 - 416\,k^2\,\gamma^4 - 419\,k^3\,\gamma^4 - 242\,k^4\,\gamma^4 - 80\,k^5\,\gamma^4 - 14\,k^6\,\gamma^4 - k^7\,\gamma^4 - 24\,k\,\gamma^k + 84\,k^3\,\gamma^k + 60\,k^4\,\gamma^k - 36\,k^5\,\gamma^k - 60\,k^6\,\gamma^k - 24\,k^7\,\gamma^k + 12\,k\,\gamma^{2\,k} + 12\,k^2\,\gamma^{2\,k} - 69\,k^3\,\gamma^{2\,k} - 168\,k^4\,\gamma^{2\,k} - 150\,k^5\,\gamma^{2\,k} - 60\,k^6\,\gamma^{2\,k} - 9\,k^7\,\gamma^{2\,k} + 48\,k\,\gamma^{1+k} + 72\,k^2\,\gamma^{1+k} - 72\,k^3\,\gamma^{1+k} + 48\,k^4\,\gamma^{1+k} + 336\,k^5\,\gamma^{1+k} + 336\,k^6\,\gamma^{1+k} + 96\,k^7\,\gamma^{1+k} - 192\,k\,\gamma^{2+k} - 552\,k^2\,\gamma^{2+k} - 432\,k^3\,\gamma^{2+k} - 528\,k^4\,\gamma^{2+k} - 960\,k^5\,\gamma^{2+k} - 648\,k^6\,\gamma^{2+k} - 144\,k^7\,\gamma^{2+k} + 144\,\gamma^{3+k} + 168\,k\,\gamma^{3+k} - 96\,k^2\,\gamma^{3+k} + 120\,k^3\,\gamma^{3+k} + 864\,k^4\,\gamma^{3+k} + 1056\,k^5\,\gamma^{3+k} + 528\,k^6\,\gamma^{3+k} + 96\,k^7\,\gamma^{3+k} + 144\,\gamma^{4+k} + 456\,k\,\gamma^{4+k} + 456\,k^2\,\gamma^{4+k} - 36\,k^3\,\gamma^{4+k} - 444\,k^4\,\gamma^{4+k} - 396\,k^5\,\gamma^{4+k} - 156\,k^6\,\gamma^{4+k} - 24\,k^7\,\gamma^{4+k} - 24\,k\,\gamma^{1+2\,k} - 60\,k^2\,\gamma^{1+2\,k} + 192\,k^3\,\gamma^{1+2\,k} + 660\,k^4\,\gamma^{1+2\,k} + 660\,k^5\,\gamma^{1+2\,k} + 264\,k^6\,\gamma^{1+2\,k} + 36\,k^7\,\gamma^{1+2\,k} + 96\,k\,\gamma^{2+2\,k} + 420\,k^2\,\gamma^{2+2\,k} + 246\,k^3\,\gamma^{2+2\,k} - 732\,k^4\,\gamma^{2+2\,k} - 984\,k^5\,\gamma^{2+2\,k} - 432\,k^6\,\gamma^{2+2\,k} - 54\,k^7\,\gamma^{2+2\,k} - 144\,\gamma^{3+2\,k} - 288\,k\,\gamma^{3+2\,k} - 252\,k^2\,\gamma^{3+2\,k} - 336\,k^3\,\gamma^{3+2\,k} + 84\,k^4\,\gamma^{3+2\,k} + 588\,k^5\,\gamma^{3+2\,k} + 312\,k^6\,\gamma^{3+2\,k} + 36\,k^7\,\gamma^{3+2\,k} - 144\,\gamma^{4+2\,k} - 252\,k\,\gamma^{4+2\,k} + 72\,k^2\,\gamma^{4+2\,k} + 375\,k^3\,\gamma^{4+2\,k} + 156\,k^4\,\gamma^{4+2\,k} - 114\,k^5\,\gamma^{4+2\,k} - 84\,k^6\,\gamma^{4+2\,k} - 9\,k^7\,\gamma^{4+2\,k} + 48\,\gamma^{3+3\,k} + 136\,k\,\gamma^{3+3\,k} + 24\,k^2\,\gamma^{3+3\,k} - 144\,k^3\,\gamma^{3+3\,k} - 64\,k^4\,\gamma^{3+3\,k} + 48\,\gamma^{4+3\,k} + 16\,k\,\gamma^{4+3\,k} - 112\,k^2\,\gamma^{4+3\,k} - 16\,k^3\,\gamma^{4+3\,k} + 64\,k^4\,\gamma^{4+3\,k}\right)$$

```
(*Define g2 as the derivative of g1 divided by the non-negative factor γ^(-2+k).
    The non-negativity of g1 and g2 for γ≥1 is equivalent.*)
```

In[35]:=
```
g2[γ_, k_] = Factor[D[g1[γ, k], γ]] / γ^(-2+k)
```

Out[35]=
$$12\,k - 12\,k^2 - 15\,k^3 + 10\,k^4 + 12\,k^5 - 6\,k^6 - k^7 - 24\,k\,\gamma - 12\,k^2\,\gamma - 24\,k^3\,\gamma - 20\,k^4\,\gamma + 44\,k^5\,\gamma + 32\,k^6\,\gamma + 4\,k^7\,\gamma + 96\,k\,\gamma^2 + 132\,k^2\,\gamma^2 - 102\,k^3\,\gamma^2 - 288\,k^4\,\gamma^2 - 204\,k^5\,\gamma^2 - 60\,k^6\,\gamma^2 - 6\,k^7\,\gamma^2 - 48\,\gamma^3 - 16\,k\,\gamma^3 + 324\,k^2\,\gamma^3 + 648\,k^3\,\gamma^3 + 540\,k^4\,\gamma^3 + 228\,k^5\,\gamma^3 + 48\,k^6\,\gamma^3 + 4\,k^7\,\gamma^3 - 48\,\gamma^4 - 220\,k\,\gamma^4 - 416\,k^2\,\gamma^4 - 419\,k^3\,\gamma^4 - 242\,k^4\,\gamma^4 - 80\,k^5\,\gamma^4 - 14\,k^6\,\gamma^4 - k^7\,\gamma^4 - 24\,k\,\gamma^k + 84\,k^3\,\gamma^k + 60\,k^4\,\gamma^k - 36\,k^5\,\gamma^k - 60\,k^6\,\gamma^k - 24\,k^7\,\gamma^k + 12\,k\,\gamma^{2\,k} + 12\,k^2\,\gamma^{2\,k} - 69\,k^3\,\gamma^{2\,k} - 168\,k^4\,\gamma^{2\,k} - 150\,k^5\,\gamma^{2\,k} - 60\,k^6\,\gamma^{2\,k} - 9\,k^7\,\gamma^{2\,k} + 48\,k\,\gamma^{1+k} + 72\,k^2\,\gamma^{1+k} - 72\,k^3\,\gamma^{1+k} + 48\,k^4\,\gamma^{1+k} + 336\,k^5\,\gamma^{1+k} + 336\,k^6\,\gamma^{1+k} + 96\,k^7\,\gamma^{1+k} - 192\,k\,\gamma^{2+k} - 552\,k^2\,\gamma^{2+k} - 432\,k^3\,\gamma^{2+k} - 528\,k^4\,\gamma^{2+k} - 960\,k^5\,\gamma^{2+k} - 648\,k^6\,\gamma^{2+k} - 144\,k^7\,\gamma^{2+k} + 144\,\gamma^{3+k} + 168\,k\,\gamma^{3+k} - 96\,k^2\,\gamma^{3+k} + 120\,k^3\,\gamma^{3+k} + 864\,k^4\,\gamma^{3+k} + 1056\,k^5\,\gamma^{3+k} + 528\,k^6\,\gamma^{3+k} + 96\,k^7\,\gamma^{3+k} + 144\,\gamma^{4+k} + 456\,k\,\gamma^{4+k} + 456\,k^2\,\gamma^{4+k} - 36\,k^3\,\gamma^{4+k} - 444\,k^4\,\gamma^{4+k} - 396\,k^5\,\gamma^{4+k} - 156\,k^6\,\gamma^{4+k} - 24\,k^7\,\gamma^{4+k} - 24\,k\,\gamma^{1+2\,k} - 60\,k^2\,\gamma^{1+2\,k} + 192\,k^3\,\gamma^{1+2\,k} + 660\,k^4\,\gamma^{1+2\,k} + 660\,k^5\,\gamma^{1+2\,k} + 264\,k^6\,\gamma^{1+2\,k} + 36\,k^7\,\gamma^{1+2\,k} + 96\,k\,\gamma^{2+2\,k} + 420\,k^2\,\gamma^{2+2\,k} + 246\,k^3\,\gamma^{2+2\,k} - 732\,k^4\,\gamma^{2+2\,k} - 984\,k^5\,\gamma^{2+2\,k} - 432\,k^6\,\gamma^{2+2\,k} - 54\,k^7\,\gamma^{2+2\,k} - 144\,\gamma^{3+2\,k} - 288\,k\,\gamma^{3+2\,k} - 252\,k^2\,\gamma^{3+2\,k} - 336\,k^3\,\gamma^{3+2\,k} + 84\,k^4\,\gamma^{3+2\,k} + 588\,k^5\,\gamma^{3+2\,k} + 312\,k^6\,\gamma^{3+2\,k} + 36\,k^7\,\gamma^{3+2\,k} - 144\,\gamma^{4+2\,k} - 252\,k\,\gamma^{4+2\,k} + 72\,k^2\,\gamma^{4+2\,k} + 375\,k^3\,\gamma^{4+2\,k} + 156\,k^4\,\gamma^{4+2\,k} - 114\,k^5\,\gamma^{4+2\,k} - 84\,k^6\,\gamma^{4+2\,k} - 9\,k^7\,\gamma^{4+2\,k} + 48\,\gamma^{3+3\,k} + 136\,k\,\gamma^{3+3\,k} + 24\,k^2\,\gamma^{3+3\,k} - 144\,k^3\,\gamma^{3+3\,k} - 64\,k^4\,\gamma^{3+3\,k} + 48\,\gamma^{4+3\,k} + 16\,k\,\gamma^{4+3\,k} - 112\,k^2\,\gamma^{4+3\,k} - 16\,k^3\,\gamma^{4+3\,k} + 64\,k^4\,\gamma^{4+3\,k}$$

```
(*Check g2[1,k]*)
```

In[36]:=
```
g2[1, k]
```

Out[36]=
```
0
```

```
(*Since g2[1,k] = 0 for any k,
it suffices to show that the derivative of g2 is non-negative for γ≥1*)

(*Factorize the derivative of g2*)
```

In[37]:=
```
Factor[D[g2[γ, k], γ]]
```

Out[37]=
$$-\frac{1}{\gamma}2\,(1+k)\,\Big(12\,k\,\gamma-6\,k^2\,\gamma+18\,k^3\,\gamma-8\,k^4\,\gamma-14\,k^5\,\gamma-2\,k^6\,\gamma-96\,k\,\gamma^2-36\,k^2\,\gamma^2+138\,k^3\,\gamma^2+150\,k^4\,\gamma^2+54\,k^5\,\gamma^2+6\,k^6\,\gamma^2+72\,\gamma^3-48\,k\,\gamma^3-438\,k^2\,\gamma^3-534\,k^3\,\gamma^3-276\,k^4\,\gamma^3-66\,k^5\,\gamma^3-6\,k^6\,\gamma^3+96\,\gamma^4+344\,k\,\gamma^4+488\,k^2\,\gamma^4+350\,k^3\,\gamma^4+134\,k^4\,\gamma^4+26\,k^5\,\gamma^4+2\,k^6\,\gamma^4+12\,k^2\,\gamma^k-12\,k^3\,\gamma^k-30\,k^4\,\gamma^k+18\,k^6\,\gamma^k+12\,k^7\,\gamma^k-12\,k^2\,\gamma^{2\,k}+69\,k^4\,\gamma^{2\,k}+99\,k^5\,\gamma^{2\,k}+51\,k^6\,\gamma^{2\,k}+9\,k^7\,\gamma^{2\,k}-24\,k\,\gamma^{1+k}-36\,k^2\,\gamma^{1+k}+36\,k^3\,\gamma^{1+k}-24\,k^4\,\gamma^{1+k}-168\,k^5\,\gamma^{1+k}-168\,k^6\,\gamma^{1+k}-48\,k^7\,\gamma^{1+k}+192\,k\,\gamma^{2+k}+456\,k^2\,\gamma^{2+k}+252\,k^3\,\gamma^{2+k}+492\,k^4\,\gamma^{2+k}+732\,k^5\,\gamma^{2+k}+396\,k^6\,\gamma^{2+k}+72\,k^7\,\gamma^{2+k}-216\,\gamma^{3+k}-108\,k\,\gamma^{3+k}+168\,k^2\,\gamma^{3+k}-300\,k^3\,\gamma^{3+k}-1056\,k^4\,\gamma^{3+k}-960\,k^5\,\gamma^{3+k}-360\,k^6\,\gamma^{3+k}-48\,k^7\,\gamma^{3+k}-288\,\gamma^{4+k}-696\,k\,\gamma^{4+k}-444\,k^2\,\gamma^{4+k}+288\,k^3\,\gamma^{4+k}+618\,k^4\,\gamma^{4+k}+396\,k^5\,\gamma^{4+k}+114\,k^6\,\gamma^{4+k}+12\,k^7\,\gamma^{4+k}+12\,k\,\gamma^{1+2\,k}+42\,k^2\,\gamma^{1+2\,k}-78\,k^3\,\gamma^{1+2\,k}-444\,k^4\,\gamma^{1+2\,k}-546\,k^5\,\gamma^{1+2\,k}-246\,k^6\,\gamma^{1+2\,k}-36\,k^7\,\gamma^{1+2\,k}-96\,k\,\gamma^{2+2\,k}-420\,k^2\,\gamma^{2+2\,k}-246\,k^3\,\gamma^{2+2\,k}+732\,k^4\,\gamma^{2+2\,k}+984\,k^5\,\gamma^{2+2\,k}+432\,k^6\,\gamma^{2+2\,k}+54\,k^7\,\gamma^{2+2\,k}+216\,\gamma^{3+2\,k}+360\,k\,\gamma^{3+2\,k}+306\,k^2\,\gamma^{3+2\,k}+450\,k^3\,\gamma^{3+2\,k}-240\,k^4\,\gamma^{3+2\,k}-726\,k^5\,\gamma^{3+2\,k}-330\,k^6\,\gamma^{3+2\,k}-36\,k^7\,\gamma^{3+2\,k}+288\,\gamma^{4+2\,k}+360\,k\,\gamma^{4+2\,k}-252\,k^2\,\gamma^{4+2\,k}-570\,k^3\,\gamma^{4+2\,k}-117\,k^4\,\gamma^{4+2\,k}+189\,k^5\,\gamma^{4+2\,k}+93\,k^6\,\gamma^{4+2\,k}+9\,k^7\,\gamma^{4+2\,k}-72\,\gamma^{3+3\,k}-204\,k\,\gamma^{3+3\,k}-36\,k^2\,\gamma^{3+3\,k}+216\,k^3\,\gamma^{3+3\,k}+96\,k^4\,\gamma^{3+3\,k}-96\,\gamma^{4+3\,k}-8\,k\,\gamma^{4+3\,k}+208\,k^2\,\gamma^{4+3\,k}-8\,k^3\,\gamma^{4+3\,k}-96\,k^4\,\gamma^{4+3\,k}\Big)$$

```
(*Define g3 as the derivative of g2 divided by the non-negative factor 2(1+k).
    The non-negativity of g2 and g3 for γ≥1 is equivalent.*)
```

In[52]:=
```
g3[γ_, k_] = Expand[Factor[D[g2[γ, k], γ]] / (2 (1 + k))]
```

Out[52]=
$$-12\,k+6\,k^2-18\,k^3+8\,k^4+14\,k^5+2\,k^6+96\,k\,\gamma+36\,k^2\,\gamma-138\,k^3\,\gamma-150\,k^4\,\gamma-54\,k^5\,\gamma-6\,k^6\,\gamma-72\,\gamma^2+48\,k\,\gamma^2+438\,k^2\,\gamma^2+534\,k^3\,\gamma^2+276\,k^4\,\gamma^2+66\,k^5\,\gamma^2+6\,k^6\,\gamma^2-96\,\gamma^3-344\,k\,\gamma^3-488\,k^2\,\gamma^3-350\,k^3\,\gamma^3-134\,k^4\,\gamma^3-26\,k^5\,\gamma^3-2\,k^6\,\gamma^3-12\,k^2\,\gamma^{-1+k}+12\,k^3\,\gamma^{-1+k}+30\,k^4\,\gamma^{-1+k}-18\,k^6\,\gamma^{-1+k}-12\,k^7\,\gamma^{-1+k}+24\,k\,\gamma^k+36\,k^2\,\gamma^k-36\,k^3\,\gamma^k+24\,k^4\,\gamma^k+168\,k^5\,\gamma^k+168\,k^6\,\gamma^k+48\,k^7\,\gamma^k-12\,k\,\gamma^{2\,k}-42\,k^2\,\gamma^{2\,k}+78\,k^3\,\gamma^{2\,k}+444\,k^4\,\gamma^{2\,k}+546\,k^5\,\gamma^{2\,k}+246\,k^6\,\gamma^{2\,k}+36\,k^7\,\gamma^{2\,k}-192\,k\,\gamma^{1+k}-456\,k^2\,\gamma^{1+k}-252\,k^3\,\gamma^{1+k}-492\,k^4\,\gamma^{1+k}-732\,k^5\,\gamma^{1+k}-396\,k^6\,\gamma^{1+k}-72\,k^7\,\gamma^{1+k}+216\,\gamma^{2+k}+108\,k\,\gamma^{2+k}-168\,k^2\,\gamma^{2+k}+300\,k^3\,\gamma^{2+k}+1056\,k^4\,\gamma^{2+k}+960\,k^5\,\gamma^{2+k}+360\,k^6\,\gamma^{2+k}+48\,k^7\,\gamma^{2+k}+288\,\gamma^{3+k}+696\,k\,\gamma^{3+k}+444\,k^2\,\gamma^{3+k}-288\,k^3\,\gamma^{3+k}-618\,k^4\,\gamma^{3+k}-396\,k^5\,\gamma^{3+k}-114\,k^6\,\gamma^{3+k}-12\,k^7\,\gamma^{3+k}+12\,k^2\,\gamma^{-1+2\,k}-69\,k^4\,\gamma^{-1+2\,k}-99\,k^5\,\gamma^{-1+2\,k}-51\,k^6\,\gamma^{-1+2\,k}-9\,k^7\,\gamma^{-1+2\,k}+96\,k\,\gamma^{1+2\,k}+420\,k^2\,\gamma^{1+2\,k}+246\,k^3\,\gamma^{1+2\,k}-732\,k^4\,\gamma^{1+2\,k}-984\,k^5\,\gamma^{1+2\,k}-432\,k^6\,\gamma^{1+2\,k}-54\,k^7\,\gamma^{1+2\,k}-216\,\gamma^{2+2\,k}-360\,k\,\gamma^{2+2\,k}-306\,k^2\,\gamma^{2+2\,k}-450\,k^3\,\gamma^{2+2\,k}+240\,k^4\,\gamma^{2+2\,k}+726\,k^5\,\gamma^{2+2\,k}+330\,k^6\,\gamma^{2+2\,k}+36\,k^7\,\gamma^{2+2\,k}-288\,\gamma^{3+2\,k}-360\,k\,\gamma^{3+2\,k}+252\,k^2\,\gamma^{3+2\,k}+570\,k^3\,\gamma^{3+2\,k}+117\,k^4\,\gamma^{3+2\,k}-189\,k^5\,\gamma^{3+2\,k}-93\,k^6\,\gamma^{3+2\,k}-9\,k^7\,\gamma^{3+2\,k}+72\,\gamma^{2+3\,k}+204\,k\,\gamma^{2+3\,k}+36\,k^2\,\gamma^{2+3\,k}-216\,k^3\,\gamma^{2+3\,k}-96\,k^4\,\gamma^{2+3\,k}+96\,\gamma^{3+3\,k}+8\,k\,\gamma^{3+3\,k}-208\,k^2\,\gamma^{3+3\,k}+8\,k^3\,\gamma^{3+3\,k}+96\,k^4\,\gamma^{3+3\,k}$$

```
(*Repeating the above procedures....*)
```

In[40]:=
```
g3[1, γ]
```

Out[40]=
```
0
```

In[41]:=
```
Factor[D[g3[γ, k], γ]]
```

Out[41]=
$$-\frac{1}{\gamma^2}3\left(-32\,k\,\gamma^2-12\,k^2\,\gamma^2+46\,k^3\,\gamma^2+50\,k^4\,\gamma^2+18\,k^5\,\gamma^2+2\,k^6\,\gamma^2+48\,\gamma^3-32\,k\,\gamma^3-292\,k^2\,\gamma^3-356\,k^3\,\gamma^3-184\,k^4\,\gamma^3-44\,k^5\,\gamma^3-4\,k^6\,\gamma^3+96\,\gamma^4+344\,k\,\gamma^4+488\,k^2\,\gamma^4+350\,k^3\,\gamma^4+134\,k^4\,\gamma^4+26\,k^5\,\gamma^4+2\,k^6\,\gamma^4-4\,k^2\,\gamma^k+8\,k^3\,\gamma^k+6\,k^4\,\gamma^k-10\,k^5\,\gamma^k-6\,k^6\,\gamma^k+2\,k^7\,\gamma^k+4\,k^8\,\gamma^k+4\,k^2\,\gamma^{2\,k}-8\,k^3\,\gamma^{2\,k}-23\,k^4\,\gamma^{2\,k}+13\,k^5\,\gamma^{2\,k}+49\,k^6\,\gamma^{2\,k}+31\,k^7\,\gamma^{2\,k}+6\,k^8\,\gamma^{2\,k}-8\,k^2\,\gamma^{1+k}-12\,k^3\,\gamma^{1+k}+12\,k^4\,\gamma^{1+k}-8\,k^5\,\gamma^{1+k}-56\,k^6\,\gamma^{1+k}-56\,k^7\,\gamma^{1+k}-16\,k^8\,\gamma^{1+k}+64\,k\,\gamma^{2+k}+216\,k^2\,\gamma^{2+k}+236\,k^3\,\gamma^{2+k}+248\,k^4\,\gamma^{2+k}+408\,k^5\,\gamma^{2+k}+376\,k^6\,\gamma^{2+k}+156\,k^7\,\gamma^{2+k}+24\,k^8\,\gamma^{2+k}-144\,\gamma^{3+k}-144\,k\,\gamma^{3+k}+76\,k^2\,\gamma^{3+k}-144\,k^3\,\gamma^{3+k}-804\,k^4\,\gamma^{3+k}-992\,k^5\,\gamma^{3+k}-560\,k^6\,\gamma^{3+k}-152\,k^7\,\gamma^{3+k}-16\,k^8\,\gamma^{3+k}-288\,\gamma^{4+k}-792\,k\,\gamma^{4+k}-676\,k^2\,\gamma^{4+k}+140\,k^3\,\gamma^{4+k}+714\,k^4\,\gamma^{4+k}+602\,k^5\,\gamma^{4+k}+246\,k^6\,\gamma^{4+k}+50\,k^7\,\gamma^{4+k}+4\,k^8\,\gamma^{4+k}+8\,k^2\,\gamma^{1+2\,k}+28\,k^3\,\gamma^{1+2\,k}-52\,k^4\,\gamma^{1+2\,k}-296\,k^5\,\gamma^{1+2\,k}-364\,k^6\,\gamma^{1+2\,k}-164\,k^7\,\gamma^{1+2\,k}-24\,k^8\,\gamma^{1+2\,k}-32\,k\,\gamma^{2+2\,k}-204\,k^2\,\gamma^{2+2\,k}-362\,k^3\,\gamma^{2+2\,k}+80\,k^4\,\gamma^{2+2\,k}+816\,k^5\,\gamma^{2+2\,k}+800\,k^6\,\gamma^{2+2\,k}+306\,k^7\,\gamma^{2+2\,k}+36\,k^8\,\gamma^{2+2\,k}+144\,\gamma^{3+2\,k}+384\,k\,\gamma^{3+2\,k}+444\,k^2\,\gamma^{3+2\,k}+504\,k^3\,\gamma^{3+2\,k}+140\,k^4\,\gamma^{3+2\,k}-644\,k^5\,\gamma^{3+2\,k}-704\,k^6\,\gamma^{3+2\,k}-244\,k^7\,\gamma^{3+2\,k}-24\,k^8\,\gamma^{3+2\,k}+288\,\gamma^{4+2\,k}+552\,k\,\gamma^{4+2\,k}-12\,k^2\,\gamma^{4+2\,k}-738\,k^3\,\gamma^{4+2\,k}-497\,k^4\,\gamma^{4+2\,k}+111\,k^5\,\gamma^{4+2\,k}+219\,k^6\,\gamma^{4+2\,k}+71\,k^7\,\gamma^{4+2\,k}+6\,k^8\,\gamma^{4+2\,k}-48\,\gamma^{3+3\,k}-208\,k\,\gamma^{3+3\,k}-228\,k^2\,\gamma^{3+3\,k}+108\,k^3\,\gamma^{3+3\,k}+280\,k^4\,\gamma^{3+3\,k}+96\,k^5\,\gamma^{3+3\,k}-96\,\gamma^{4+3\,k}-104\,k\,\gamma^{4+3\,k}+200\,k^2\,\gamma^{4+3\,k}+200\,k^3\,\gamma^{4+3\,k}-104\,k^4\,\gamma^{4+3\,k}-96\,k^5\,\gamma^{4+3\,k}\right)$$

In[50]:=
```
g4[γ_, k_] = Expand[D[g3[γ, k], γ] / (3)]
```

Out[50]=
$$32\,k+12\,k^2-46\,k^3-50\,k^4-18\,k^5-2\,k^6-48\,\gamma+32\,k\,\gamma+292\,k^2\,\gamma+356\,k^3\,\gamma+184\,k^4\,\gamma+44\,k^5\,\gamma+4\,k^6\,\gamma-96\,\gamma^2-344\,k\,\gamma^2-488\,k^2\,\gamma^2-350\,k^3\,\gamma^2-134\,k^4\,\gamma^2-26\,k^5\,\gamma^2-2\,k^6\,\gamma^2+4\,k^2\,\gamma^{-2+k}-8\,k^3\,\gamma^{-2+k}-6\,k^4\,\gamma^{-2+k}+10\,k^5\,\gamma^{-2+k}+6\,k^6\,\gamma^{-2+k}-2\,k^7\,\gamma^{-2+k}-4\,k^8\,\gamma^{-2+k}+8\,k^2\,\gamma^{-1+k}+12\,k^3\,\gamma^{-1+k}-12\,k^4\,\gamma^{-1+k}+8\,k^5\,\gamma^{-1+k}+56\,k^6\,\gamma^{-1+k}+56\,k^7\,\gamma^{-1+k}+16\,k^8\,\gamma^{-1+k}-64\,k\,\gamma^k-216\,k^2\,\gamma^k-236\,k^3\,\gamma^k-248\,k^4\,\gamma^k-408\,k^5\,\gamma^k-376\,k^6\,\gamma^k-156\,k^7\,\gamma^k-24\,k^8\,\gamma^k+32\,k\,\gamma^{2\,k}+204\,k^2\,\gamma^{2\,k}+362\,k^3\,\gamma^{2\,k}-80\,k^4\,\gamma^{2\,k}-816\,k^5\,\gamma^{2\,k}-800\,k^6\,\gamma^{2\,k}-306\,k^7\,\gamma^{2\,k}-36\,k^8\,\gamma^{2\,k}+144\,\gamma^{1+k}+144\,k\,\gamma^{1+k}-76\,k^2\,\gamma^{1+k}+144\,k^3\,\gamma^{1+k}+804\,k^4\,\gamma^{1+k}+992\,k^5\,\gamma^{1+k}+560\,k^6\,\gamma^{1+k}+152\,k^7\,\gamma^{1+k}+16\,k^8\,\gamma^{1+k}+288\,\gamma^{2+k}+792\,k\,\gamma^{2+k}+676\,k^2\,\gamma^{2+k}-140\,k^3\,\gamma^{2+k}-714\,k^4\,\gamma^{2+k}-602\,k^5\,\gamma^{2+k}-246\,k^6\,\gamma^{2+k}-50\,k^7\,\gamma^{2+k}-4\,k^8\,\gamma^{2+k}-4\,k^2\,\gamma^{-2+2\,k}+8\,k^3\,\gamma^{-2+2\,k}+23\,k^4\,\gamma^{-2+2\,k}-13\,k^5\,\gamma^{-2+2\,k}-49\,k^6\,\gamma^{-2+2\,k}-31\,k^7\,\gamma^{-2+2\,k}-6\,k^8\,\gamma^{-2+2\,k}-8\,k^2\,\gamma^{-1+2\,k}-28\,k^3\,\gamma^{-1+2\,k}+52\,k^4\,\gamma^{-1+2\,k}+296\,k^5\,\gamma^{-1+2\,k}+364\,k^6\,\gamma^{-1+2\,k}+164\,k^7\,\gamma^{-1+2\,k}+24\,k^8\,\gamma^{-1+2\,k}-144\,\gamma^{1+2\,k}-384\,k\,\gamma^{1+2\,k}-444\,k^2\,\gamma^{1+2\,k}-504\,k^3\,\gamma^{1+2\,k}-140\,k^4\,\gamma^{1+2\,k}+644\,k^5\,\gamma^{1+2\,k}+704\,k^6\,\gamma^{1+2\,k}+244\,k^7\,\gamma^{1+2\,k}+24\,k^8\,\gamma^{1+2\,k}-288\,\gamma^{2+2\,k}-552\,k\,\gamma^{2+2\,k}+12\,k^2\,\gamma^{2+2\,k}+738\,k^3\,\gamma^{2+2\,k}+497\,k^4\,\gamma^{2+2\,k}-111\,k^5\,\gamma^{2+2\,k}-219\,k^6\,\gamma^{2+2\,k}-71\,k^7\,\gamma^{2+2\,k}-6\,k^8\,\gamma^{2+2\,k}+48\,\gamma^{1+3\,k}+208\,k\,\gamma^{1+3\,k}+228\,k^2\,\gamma^{1+3\,k}-108\,k^3\,\gamma^{1+3\,k}-280\,k^4\,\gamma^{1+3\,k}-96\,k^5\,\gamma^{1+3\,k}+96\,\gamma^{2+3\,k}+104\,k\,\gamma^{2+3\,k}-200\,k^2\,\gamma^{2+3\,k}-200\,k^3\,\gamma^{2+3\,k}+104\,k^4\,\gamma^{2+3\,k}+96\,k^5\,\gamma^{2+3\,k}$$

In[43]:=
```
g[1, k]
```

Out[43]=
```
0
```

In[44]:=
```
Factor[D[g4[γ, k], γ]]
```

Out[44]=
$$-\frac{1}{\gamma^3}2\,\left(24\,\gamma^3-16\,k\,\gamma^3-146\,k^2\,\gamma^3-178\,k^3\,\gamma^3-92\,k^4\,\gamma^3-22\,k^5\,\gamma^3-2\,k^6\,\gamma^3+96\,\gamma^4+344\,k\,\gamma^4+488\,k^2\,\gamma^4+\right.$$
$$350\,k^3\,\gamma^4+134\,k^4\,\gamma^4+26\,k^5\,\gamma^4+2\,k^6\,\gamma^4+4\,k^2\,\gamma^k-10\,k^3\,\gamma^k-2\,k^4\,\gamma^k+13\,k^5\,\gamma^k+k^6\,\gamma^k-5\,k^7\,\gamma^k-$$
$$3\,k^8\,\gamma^k+2\,k^9\,\gamma^k-4\,k^2\,\gamma^{2\,k}+12\,k^3\,\gamma^{2\,k}+15\,k^4\,\gamma^{2\,k}-36\,k^5\,\gamma^{2\,k}-36\,k^6\,\gamma^{2\,k}+18\,k^7\,\gamma^{2\,k}+25\,k^8\,\gamma^{2\,k}+$$
$$6\,k^9\,\gamma^{2\,k}+4\,k^2\,\gamma^{1+k}+2\,k^3\,\gamma^{1+k}-12\,k^4\,\gamma^{1+k}+10\,k^5\,\gamma^{1+k}+24\,k^6\,\gamma^{1+k}-20\,k^8\,\gamma^{1+k}-8\,k^9\,\gamma^{1+k}+$$
$$32\,k^2\,\gamma^{2+k}+108\,k^3\,\gamma^{2+k}+118\,k^4\,\gamma^{2+k}+124\,k^5\,\gamma^{2+k}+204\,k^6\,\gamma^{2+k}+188\,k^7\,\gamma^{2+k}+78\,k^8\,\gamma^{2+k}+$$
$$12\,k^9\,\gamma^{2+k}-72\,\gamma^{3+k}-144\,k\,\gamma^{3+k}-34\,k^2\,\gamma^{3+k}-34\,k^3\,\gamma^{3+k}-474\,k^4\,\gamma^{3+k}-898\,k^5\,\gamma^{3+k}-776\,k^6\,\gamma^{3+k}-$$
$$356\,k^7\,\gamma^{3+k}-84\,k^8\,\gamma^{3+k}-8\,k^9\,\gamma^{3+k}-288\,\gamma^{4+k}-936\,k\,\gamma^{4+k}-1072\,k^2\,\gamma^{4+k}-198\,k^3\,\gamma^{4+k}+$$
$$784\,k^4\,\gamma^{4+k}+959\,k^5\,\gamma^{4+k}+547\,k^6\,\gamma^{4+k}+173\,k^7\,\gamma^{4+k}+29\,k^8\,\gamma^{4+k}+2\,k^9\,\gamma^{4+k}-4\,k^2\,\gamma^{1+2\,k}-$$
$$6\,k^3\,\gamma^{1+2\,k}+54\,k^4\,\gamma^{1+2\,k}+96\,k^5\,\gamma^{1+2\,k}-114\,k^6\,\gamma^{1+2\,k}-282\,k^7\,\gamma^{1+2\,k}-152\,k^8\,\gamma^{1+2\,k}-24\,k^9\,\gamma^{1+2\,k}-$$
$$32\,k^2\,\gamma^{2+2\,k}-204\,k^3\,\gamma^{2+2\,k}-362\,k^4\,\gamma^{2+2\,k}+80\,k^5\,\gamma^{2+2\,k}+816\,k^6\,\gamma^{2+2\,k}+800\,k^7\,\gamma^{2+2\,k}+306\,k^8\,\gamma^{2+2\,k}+$$
$$36\,k^9\,\gamma^{2+2\,k}+72\,\gamma^{3+2\,k}+336\,k\,\gamma^{3+2\,k}+606\,k^2\,\gamma^{3+2\,k}+696\,k^3\,\gamma^{3+2\,k}+574\,k^4\,\gamma^{3+2\,k}-182\,k^5\,\gamma^{3+2\,k}-$$
$$996\,k^6\,\gamma^{3+2\,k}-826\,k^7\,\gamma^{3+2\,k}-256\,k^8\,\gamma^{3+2\,k}-24\,k^9\,\gamma^{3+2\,k}+288\,\gamma^{4+2\,k}+840\,k\,\gamma^{4+2\,k}+540\,k^2\,\gamma^{4+2\,k}-$$
$$750\,k^3\,\gamma^{4+2\,k}-1235\,k^4\,\gamma^{4+2\,k}-386\,k^5\,\gamma^{4+2\,k}+330\,k^6\,\gamma^{4+2\,k}+290\,k^7\,\gamma^{4+2\,k}+77\,k^8\,\gamma^{4+2\,k}+6\,k^9\,\gamma^{4+2\,k}-$$
$$24\,\gamma^{3+3\,k}-176\,k\,\gamma^{3+3\,k}-426\,k^2\,\gamma^{3+3\,k}-288\,k^3\,\gamma^{3+3\,k}+302\,k^4\,\gamma^{3+3\,k}+468\,k^5\,\gamma^{3+3\,k}+144\,k^6\,\gamma^{3+3\,k}-$$
$$\left.96\,\gamma^{4+3\,k}-248\,k\,\gamma^{4+3\,k}+44\,k^2\,\gamma^{4+3\,k}+500\,k^3\,\gamma^{4+3\,k}+196\,k^4\,\gamma^{4+3\,k}-252\,k^5\,\gamma^{4+3\,k}-144\,k^6\,\gamma^{4+3\,k}\right)$$

In[49]:=
```
g5[γ_, k_] = Expand[D[g4[γ, k], γ] / (2)]
```

Out[49]=
$$-24+16\,k+146\,k^2+178\,k^3+92\,k^4+22\,k^5+2\,k^6-96\,\gamma-344\,k\,\gamma-488\,k^2\,\gamma-350\,k^3\,\gamma-134\,k^4\,\gamma-$$
$$26\,k^5\,\gamma-2\,k^6\,\gamma-4\,k^2\,\gamma^{-3+k}+10\,k^3\,\gamma^{-3+k}+2\,k^4\,\gamma^{-3+k}-13\,k^5\,\gamma^{-3+k}-k^6\,\gamma^{-3+k}+5\,k^7\,\gamma^{-3+k}+$$
$$3\,k^8\,\gamma^{-3+k}-2\,k^9\,\gamma^{-3+k}-4\,k^2\,\gamma^{-2+k}-2\,k^3\,\gamma^{-2+k}+12\,k^4\,\gamma^{-2+k}-10\,k^5\,\gamma^{-2+k}-24\,k^6\,\gamma^{-2+k}+20\,k^8\,\gamma^{-2+k}+$$
$$8\,k^9\,\gamma^{-2+k}-32\,k^2\,\gamma^{-1+k}-108\,k^3\,\gamma^{-1+k}-118\,k^4\,\gamma^{-1+k}-124\,k^5\,\gamma^{-1+k}-204\,k^6\,\gamma^{-1+k}-188\,k^7\,\gamma^{-1+k}-$$
$$78\,k^8\,\gamma^{-1+k}-12\,k^9\,\gamma^{-1+k}+72\,\gamma^k+144\,k\,\gamma^k+34\,k^2\,\gamma^k+34\,k^3\,\gamma^k+474\,k^4\,\gamma^k+898\,k^5\,\gamma^k+776\,k^6\,\gamma^k+$$
$$356\,k^7\,\gamma^k+84\,k^8\,\gamma^k+8\,k^9\,\gamma^k-72\,\gamma^{2\,k}-336\,k\,\gamma^{2\,k}-606\,k^2\,\gamma^{2\,k}-696\,k^3\,\gamma^{2\,k}-574\,k^4\,\gamma^{2\,k}+$$
$$182\,k^5\,\gamma^{2\,k}+996\,k^6\,\gamma^{2\,k}+826\,k^7\,\gamma^{2\,k}+256\,k^8\,\gamma^{2\,k}+24\,k^9\,\gamma^{2\,k}+24\,\gamma^{3\,k}+176\,k\,\gamma^{3\,k}+426\,k^2\,\gamma^{3\,k}+$$
$$288\,k^3\,\gamma^{3\,k}-302\,k^4\,\gamma^{3\,k}-468\,k^5\,\gamma^{3\,k}-144\,k^6\,\gamma^{3\,k}+288\,\gamma^{1+k}+936\,k\,\gamma^{1+k}+1072\,k^2\,\gamma^{1+k}+$$
$$198\,k^3\,\gamma^{1+k}-784\,k^4\,\gamma^{1+k}-959\,k^5\,\gamma^{1+k}-547\,k^6\,\gamma^{1+k}-173\,k^7\,\gamma^{1+k}-29\,k^8\,\gamma^{1+k}-2\,k^9\,\gamma^{1+k}+$$
$$4\,k^2\,\gamma^{-3+2\,k}-12\,k^3\,\gamma^{-3+2\,k}-15\,k^4\,\gamma^{-3+2\,k}+36\,k^5\,\gamma^{-3+2\,k}+36\,k^6\,\gamma^{-3+2\,k}-18\,k^7\,\gamma^{-3+2\,k}-25\,k^8\,\gamma^{-3+2\,k}-$$
$$6\,k^9\,\gamma^{-3+2\,k}+4\,k^2\,\gamma^{-2+2\,k}+6\,k^3\,\gamma^{-2+2\,k}-54\,k^4\,\gamma^{-2+2\,k}-96\,k^5\,\gamma^{-2+2\,k}+114\,k^6\,\gamma^{-2+2\,k}+282\,k^7\,\gamma^{-2+2\,k}+$$
$$152\,k^8\,\gamma^{-2+2\,k}+24\,k^9\,\gamma^{-2+2\,k}+32\,k^2\,\gamma^{-1+2\,k}+204\,k^3\,\gamma^{-1+2\,k}+362\,k^4\,\gamma^{-1+2\,k}-80\,k^5\,\gamma^{-1+2\,k}-$$
$$816\,k^6\,\gamma^{-1+2\,k}-800\,k^7\,\gamma^{-1+2\,k}-306\,k^8\,\gamma^{-1+2\,k}-36\,k^9\,\gamma^{-1+2\,k}-288\,\gamma^{1+2\,k}-840\,k\,\gamma^{1+2\,k}-540\,k^2\,\gamma^{1+2\,k}+$$
$$750\,k^3\,\gamma^{1+2\,k}+1235\,k^4\,\gamma^{1+2\,k}+386\,k^5\,\gamma^{1+2\,k}-330\,k^6\,\gamma^{1+2\,k}-290\,k^7\,\gamma^{1+2\,k}-77\,k^8\,\gamma^{1+2\,k}-6\,k^9\,\gamma^{1+2\,k}+$$
$$96\,\gamma^{1+3\,k}+248\,k\,\gamma^{1+3\,k}-44\,k^2\,\gamma^{1+3\,k}-500\,k^3\,\gamma^{1+3\,k}-196\,k^4\,\gamma^{1+3\,k}+252\,k^5\,\gamma^{1+3\,k}+144\,k^6\,\gamma^{1+3\,k}$$

In[46]:=
```
g5[1, k]
```

Out[46]=
```
0
```

In[47]:=
```
Factor[D[g5[γ, k], γ]]
```

Out[47]=
$$-\frac{1}{\gamma^4}\Big(96\,\gamma^4+344\,k\,\gamma^4+488\,k^2\,\gamma^4+350\,k^3\,\gamma^4+134\,k^4\,\gamma^4+26\,k^5\,\gamma^4+2\,k^6\,\gamma^4-12\,k^2\,\gamma^k+34\,k^3\,\gamma^k-4\,k^4\,\gamma^k-41\,k^5\,\gamma^k+10\,k^6\,\gamma^k+16\,k^7\,\gamma^k+4\,k^8\,\gamma^k-9\,k^9\,\gamma^k+2\,k^{10}\,\gamma^k+12\,k^2\,\gamma^{2\,k}-44\,k^3\,\gamma^{2\,k}-21\,k^4\,\gamma^{2\,k}+138\,k^5\,\gamma^{2\,k}+36\,k^6\,\gamma^{2\,k}-126\,k^7\,\gamma^{2\,k}-39\,k^8\,\gamma^{2\,k}+32\,k^9\,\gamma^{2\,k}+12\,k^{10}\,\gamma^{2\,k}-8\,k^2\,\gamma^{1+k}+26\,k^4\,\gamma^{1+k}-32\,k^5\,\gamma^{1+k}-38\,k^6\,\gamma^{1+k}+24\,k^7\,\gamma^{1+k}+40\,k^8\,\gamma^{1+k}-4\,k^9\,\gamma^{1+k}-8\,k^{10}\,\gamma^{1+k}-32\,k^2\,\gamma^{2+k}-76\,k^3\,\gamma^{2+k}-10\,k^4\,\gamma^{2+k}-6\,k^5\,\gamma^{2+k}-80\,k^6\,\gamma^{2+k}+16\,k^7\,\gamma^{2+k}+110\,k^8\,\gamma^{2+k}+66\,k^9\,\gamma^{2+k}+12\,k^{10}\,\gamma^{2+k}-72\,k\,\gamma^{3+k}-144\,k^2\,\gamma^{3+k}-34\,k^3\,\gamma^{3+k}-34\,k^4\,\gamma^{3+k}-474\,k^5\,\gamma^{3+k}-898\,k^6\,\gamma^{3+k}-776\,k^7\,\gamma^{3+k}-356\,k^8\,\gamma^{3+k}-84\,k^9\,\gamma^{3+k}-8\,k^{10}\,\gamma^{3+k}-288\,\gamma^{4+k}-1224\,k\,\gamma^{4+k}-2008\,k^2\,\gamma^{4+k}-1270\,k^3\,\gamma^{4+k}+586\,k^4\,\gamma^{4+k}+1743\,k^5\,\gamma^{4+k}+1506\,k^6\,\gamma^{4+k}+720\,k^7\,\gamma^{4+k}+202\,k^8\,\gamma^{4+k}+31\,k^9\,\gamma^{4+k}+2\,k^{10}\,\gamma^{4+k}+8\,k^2\,\gamma^{1+2\,k}+4\,k^3\,\gamma^{1+2\,k}-120\,k^4\,\gamma^{1+2\,k}-84\,k^5\,\gamma^{1+2\,k}+420\,k^6\,\gamma^{1+2\,k}+336\,k^7\,\gamma^{1+2\,k}-260\,k^8\,\gamma^{1+2\,k}-256\,k^9\,\gamma^{1+2\,k}-48\,k^{10}\,\gamma^{1+2\,k}+32\,k^2\,\gamma^{2+2\,k}+140\,k^3\,\gamma^{2+2\,k}-46\,k^4\,\gamma^{2+2\,k}-804\,k^5\,\gamma^{2+2\,k}-656\,k^6\,\gamma^{2+2\,k}+832\,k^7\,\gamma^{2+2\,k}+1294\,k^8\,\gamma^{2+2\,k}+576\,k^9\,\gamma^{2+2\,k}+72\,k^{10}\,\gamma^{2+2\,k}+144\,k\,\gamma^{3+2\,k}+672\,k^2\,\gamma^{3+2\,k}+1212\,k^3\,\gamma^{3+2\,k}+1392\,k^4\,\gamma^{3+2\,k}+1148\,k^5\,\gamma^{3+2\,k}-364\,k^6\,\gamma^{3+2\,k}-1992\,k^7\,\gamma^{3+2\,k}-1652\,k^8\,\gamma^{3+2\,k}-512\,k^9\,\gamma^{3+2\,k}-48\,k^{10}\,\gamma^{3+2\,k}+288\,\gamma^{4+2\,k}+1416\,k\,\gamma^{4+2\,k}+2220\,k^2\,\gamma^{4+2\,k}+330\,k^3\,\gamma^{4+2\,k}-2735\,k^4\,\gamma^{4+2\,k}-2856\,k^5\,\gamma^{4+2\,k}-442\,k^6\,\gamma^{4+2\,k}+950\,k^7\,\gamma^{4+2\,k}+657\,k^8\,\gamma^{4+2\,k}+160\,k^9\,\gamma^{4+2\,k}+12\,k^{10}\,\gamma^{4+2\,k}-72\,k\,\gamma^{3+3\,k}-528\,k^2\,\gamma^{3+3\,k}-1278\,k^3\,\gamma^{3+3\,k}-864\,k^4\,\gamma^{3+3\,k}+906\,k^5\,\gamma^{3+3\,k}+1404\,k^6\,\gamma^{3+3\,k}+432\,k^7\,\gamma^{3+3\,k}-96\,\gamma^{4+3\,k}-536\,k\,\gamma^{4+3\,k}-700\,k^2\,\gamma^{4+3\,k}+632\,k^3\,\gamma^{4+3\,k}+1696\,k^4\,\gamma^{4+3\,k}+336\,k^5\,\gamma^{4+3\,k}-900\,k^6\,\gamma^{4+3\,k}-432\,k^7\,\gamma^{4+3\,k}\Big)$$

In[48]:=
```
g6[γ_, k_] = D[g5[γ, k], γ]
```

Out[48]=
$$-96-344\,k-488\,k^2-350\,k^3-134\,k^4-26\,k^5-2\,k^6-4\,(-3+k)\,k^2\,\gamma^{-4+k}+10\,(-3+k)\,k^3\,\gamma^{-4+k}+2\,(-3+k)\,k^4\,\gamma^{-4+k}-13\,(-3+k)\,k^5\,\gamma^{-4+k}-(-3+k)\,k^6\,\gamma^{-4+k}+5\,(-3+k)\,k^7\,\gamma^{-4+k}+3\,(-3+k)\,k^8\,\gamma^{-4+k}-2\,(-3+k)\,k^9\,\gamma^{-4+k}-4\,(-2+k)\,k^2\,\gamma^{-3+k}-2\,(-2+k)\,k^3\,\gamma^{-3+k}+12\,(-2+k)\,k^4\,\gamma^{-3+k}-10\,(-2+k)\,k^5\,\gamma^{-3+k}-24\,(-2+k)\,k^6\,\gamma^{-3+k}+20\,(-2+k)\,k^8\,\gamma^{-3+k}+8\,(-2+k)\,k^9\,\gamma^{-3+k}-32\,(-1+k)\,k^2\,\gamma^{-2+k}-108\,(-1+k)\,k^3\,\gamma^{-2+k}-118\,(-1+k)\,k^4\,\gamma^{-2+k}-124\,(-1+k)\,k^5\,\gamma^{-2+k}-204\,(-1+k)\,k^6\,\gamma^{-2+k}-188\,(-1+k)\,k^7\,\gamma^{-2+k}-78\,(-1+k)\,k^8\,\gamma^{-2+k}-12\,(-1+k)\,k^9\,\gamma^{-2+k}+72\,k\,\gamma^{-1+k}+144\,k^2\,\gamma^{-1+k}+34\,k^3\,\gamma^{-1+k}+34\,k^4\,\gamma^{-1+k}+474\,k^5\,\gamma^{-1+k}+898\,k^6\,\gamma^{-1+k}+776\,k^7\,\gamma^{-1+k}+356\,k^8\,\gamma^{-1+k}+84\,k^9\,\gamma^{-1+k}+8\,k^{10}\,\gamma^{-1+k}+288\,(1+k)\,\gamma^k+936\,k\,(1+k)\,\gamma^k+1072\,k^2\,(1+k)\,\gamma^k+198\,k^3\,(1+k)\,\gamma^k-784\,k^4\,(1+k)\,\gamma^k-959\,k^5\,(1+k)\,\gamma^k-547\,k^6\,(1+k)\,\gamma^k-173\,k^7\,(1+k)\,\gamma^k-29\,k^8\,(1+k)\,\gamma^k-2\,k^9\,(1+k)\,\gamma^k-288\,(1+2\,k)\,\gamma^{2\,k}-840\,k\,(1+2\,k)\,\gamma^{2\,k}-540\,k^2\,(1+2\,k)\,\gamma^{2\,k}+750\,k^3\,(1+2\,k)\,\gamma^{2\,k}+1235\,k^4\,(1+2\,k)\,\gamma^{2\,k}+386\,k^5\,(1+2\,k)\,\gamma^{2\,k}-330\,k^6\,(1+2\,k)\,\gamma^{2\,k}-290\,k^7\,(1+2\,k)\,\gamma^{2\,k}-77\,k^8\,(1+2\,k)\,\gamma^{2\,k}-6\,k^9\,(1+2\,k)\,\gamma^{2\,k}+96\,(1+3\,k)\,\gamma^{3\,k}+248\,k\,(1+3\,k)\,\gamma^{3\,k}-44\,k^2\,(1+3\,k)\,\gamma^{3\,k}-500\,k^3\,(1+3\,k)\,\gamma^{3\,k}-196\,k^4\,(1+3\,k)\,\gamma^{3\,k}+252\,k^5\,(1+3\,k)\,\gamma^{3\,k}+144\,k^6\,(1+3\,k)\,\gamma^{3\,k}+4\,k^2\,(-3+2\,k)\,\gamma^{-4+2\,k}-12\,k^3\,(-3+2\,k)\,\gamma^{-4+2\,k}-15\,k^4\,(-3+2\,k)\,\gamma^{-4+2\,k}+36\,k^5\,(-3+2\,k)\,\gamma^{-4+2\,k}+36\,k^6\,(-3+2\,k)\,\gamma^{-4+2\,k}-18\,k^7\,(-3+2\,k)\,\gamma^{-4+2\,k}-25\,k^8\,(-3+2\,k)\,\gamma^{-4+2\,k}-6\,k^9\,(-3+2\,k)\,\gamma^{-4+2\,k}+4\,k^2\,(-2+2\,k)\,\gamma^{-3+2\,k}+6\,k^3\,(-2+2\,k)\,\gamma^{-3+2\,k}-54\,k^4\,(-2+2\,k)\,\gamma^{-3+2\,k}-96\,k^5\,(-2+2\,k)\,\gamma^{-3+2\,k}+114\,k^6\,(-2+2\,k)\,\gamma^{-3+2\,k}+282\,k^7\,(-2+2\,k)\,\gamma^{-3+2\,k}+152\,k^8\,(-2+2\,k)\,\gamma^{-3+2\,k}+24\,k^9\,(-2+2\,k)\,\gamma^{-3+2\,k}+32\,k^2\,(-1+2\,k)\,\gamma^{-2+2\,k}+204\,k^3\,(-1+2\,k)\,\gamma^{-2+2\,k}+362\,k^4\,(-1+2\,k)\,\gamma^{-2+2\,k}-80\,k^5\,(-1+2\,k)\,\gamma^{-2+2\,k}-816\,k^6\,(-1+2\,k)\,\gamma^{-2+2\,k}-800\,k^7\,(-1+2\,k)\,\gamma^{-2+2\,k}-306\,k^8\,(-1+2\,k)\,\gamma^{-2+2\,k}-36\,k^9\,(-1+2\,k)\,\gamma^{-2+2\,k}-144\,k\,\gamma^{-1+2\,k}-672\,k^2\,\gamma^{-1+2\,k}-1212\,k^3\,\gamma^{-1+2\,k}-1392\,k^4\,\gamma^{-1+2\,k}-1148\,k^5\,\gamma^{-1+2\,k}+364\,k^6\,\gamma^{-1+2\,k}+1992\,k^7\,\gamma^{-1+2\,k}+1652\,k^8\,\gamma^{-1+2\,k}+512\,k^9\,\gamma^{-1+2\,k}+48\,k^{10}\,\gamma^{-1+2\,k}+72\,k\,\gamma^{-1+3\,k}+528\,k^2\,\gamma^{-1+3\,k}+1278\,k^3\,\gamma^{-1+3\,k}+864\,k^4\,\gamma^{-1+3\,k}-906\,k^5\,\gamma^{-1+3\,k}-1404\,k^6\,\gamma^{-1+3\,k}-432\,k^7\,\gamma^{-1+3\,k}$$

In[54]:=
```
Simplify[g6[1, k]]
```

Out[54]=
```
0
```

In[55]:=
```
Factor[D[g6[γ, k], γ]]
```

Out[55]=
$$\left(-1+k\right)\, k\, \gamma^{-5+k}$$
$$\left(48\, k-100\, k^2-50\, k^3+110\, k^4+29\, k^5-25\, k^6-25\, k^7+15\, k^8-2\, k^9+24\, k\, \gamma+16\, k^2\, \gamma-62\, k^3\, \gamma+60\, k^4\, \gamma+\right.$$
$$142\, k^5\, \gamma+32\, k^6\, \gamma-64\, k^7\, \gamma-12\, k^8\, \gamma+8\, k^9\, \gamma+64\, k\, \gamma^2+184\, k^2\, \gamma^2+128\, k^3\, \gamma^2+130\, k^4\, \gamma^2+284\, k^5\, \gamma^2+$$
$$172\, k^6\, \gamma^2-32\, k^7\, \gamma^2-54\, k^8\, \gamma^2-12\, k^9\, \gamma^2+72\, \gamma^3+144\, k\, \gamma^3+34\, k^2\, \gamma^3+34\, k^3\, \gamma^3+474\, k^4\, \gamma^3+$$
$$898\, k^5\, \gamma^3+776\, k^6\, \gamma^3+356\, k^7\, \gamma^3+84\, k^8\, \gamma^3+8\, k^9\, \gamma^3-288\, \gamma^4-1512\, k\, \gamma^4-3520\, k^2\, \gamma^4-4790\, k^3\, \gamma^4-$$
$$4204\, k^4\, \gamma^4-2461\, k^5\, \gamma^4-955\, k^6\, \gamma^4-235\, k^7\, \gamma^4-33\, k^8\, \gamma^4-2\, k^9\, \gamma^4-48\, k\, \gamma^k+152\, k^2\, \gamma^k+148\, k^3\, \gamma^k-$$
$$446\, k^4\, \gamma^k-314\, k^5\, \gamma^k+262\, k^6\, \gamma^k+166\, k^7\, \gamma^k-40\, k^8\, \gamma^k-24\, k^9\, \gamma^k-24\, k\, \gamma^{1+k}-20\, k^2\, \gamma^{1+k}+348\, k^3\, \gamma^{1+k}+$$
$$360\, k^4\, \gamma^{1+k}-1068\, k^5\, \gamma^{1+k}-1236\, k^6\, \gamma^{1+k}+216\, k^7\, \gamma^{1+k}+464\, k^8\, \gamma^{1+k}+96\, k^9\, \gamma^{1+k}-64\, k\, \gamma^{2+k}-280\, k^2\, \gamma^{2+k}+$$
$$92\, k^3\, \gamma^{2+k}+1608\, k^4\, \gamma^{2+k}+1312\, k^5\, \gamma^{2+k}-1664\, k^6\, \gamma^{2+k}-2588\, k^7\, \gamma^{2+k}-1152\, k^8\, \gamma^{2+k}-144\, k^9\, \gamma^{2+k}-$$
$$144\, \gamma^{3+k}-528\, k\, \gamma^{3+k}-396\, k^2\, \gamma^{3+k}+636\, k^3\, \gamma^{3+k}+2272\, k^4\, \gamma^{3+k}+4932\, k^5\, \gamma^{3+k}+6196\, k^6\, \gamma^{3+k}+$$
$$3864\, k^7\, \gamma^{3+k}+1072\, k^8\, \gamma^{3+k}+96\, k^9\, \gamma^{3+k}+576\, \gamma^{4+k}+3408\, k\, \gamma^{4+k}+7848\, k^2\, \gamma^{4+k}+8508\, k^3\, \gamma^{4+k}+$$
$$3038\, k^4\, \gamma^{4+k}-2674\, k^5\, \gamma^{4+k}-3558\, k^6\, \gamma^{4+k}-1658\, k^7\, \gamma^{4+k}-344\, k^8\, \gamma^{4+k}-24\, k^9\, \gamma^{4+k}+72\, \gamma^{3+2\,k}+$$
$$384\, k\, \gamma^{3+2\,k}+78\, k^2\, \gamma^{3+2\,k}-2892\, k^3\, \gamma^{3+2\,k}-6390\, k^4\, \gamma^{3+2\,k}-5076\, k^5\, \gamma^{3+2\,k}-1296\, k^6\, \gamma^{3+2\,k}-288\, \gamma^{4+2\,k}-$$
$$\left.1896\, k\, \gamma^{4+2\,k}-3996\, k^2\, \gamma^{4+2\,k}-2100\, k^3\, \gamma^{4+2\,k}+2988\, k^4\, \gamma^{4+2\,k}+3996\, k^5\, \gamma^{4+2\,k}+1296\, k^6\, \gamma^{4+2\,k}\right)$$

In[57]:=
```
g7[γ_, k_] = Expand[Factor[D[g6[γ, k], γ]] / ((-1 + k) k γ^(-5+k))]
```

Out[57]=
$$48\, k-100\, k^2-50\, k^3+110\, k^4+29\, k^5-25\, k^6-25\, k^7+15\, k^8-2\, k^9+24\, k\, \gamma+16\, k^2\, \gamma-62\, k^3\, \gamma+60\, k^4\, \gamma+$$
$$142\, k^5\, \gamma+32\, k^6\, \gamma-64\, k^7\, \gamma-12\, k^8\, \gamma+8\, k^9\, \gamma+64\, k\, \gamma^2+184\, k^2\, \gamma^2+128\, k^3\, \gamma^2+130\, k^4\, \gamma^2+284\, k^5\, \gamma^2+$$
$$172\, k^6\, \gamma^2-32\, k^7\, \gamma^2-54\, k^8\, \gamma^2-12\, k^9\, \gamma^2+72\, \gamma^3+144\, k\, \gamma^3+34\, k^2\, \gamma^3+34\, k^3\, \gamma^3+474\, k^4\, \gamma^3+$$
$$898\, k^5\, \gamma^3+776\, k^6\, \gamma^3+356\, k^7\, \gamma^3+84\, k^8\, \gamma^3+8\, k^9\, \gamma^3-288\, \gamma^4-1512\, k\, \gamma^4-3520\, k^2\, \gamma^4-4790\, k^3\, \gamma^4-$$
$$4204\, k^4\, \gamma^4-2461\, k^5\, \gamma^4-955\, k^6\, \gamma^4-235\, k^7\, \gamma^4-33\, k^8\, \gamma^4-2\, k^9\, \gamma^4-48\, k\, \gamma^k+152\, k^2\, \gamma^k+148\, k^3\, \gamma^k-$$
$$446\, k^4\, \gamma^k-314\, k^5\, \gamma^k+262\, k^6\, \gamma^k+166\, k^7\, \gamma^k-40\, k^8\, \gamma^k-24\, k^9\, \gamma^k-24\, k\, \gamma^{1+k}-20\, k^2\, \gamma^{1+k}+348\, k^3\, \gamma^{1+k}+$$
$$360\, k^4\, \gamma^{1+k}-1068\, k^5\, \gamma^{1+k}-1236\, k^6\, \gamma^{1+k}+216\, k^7\, \gamma^{1+k}+464\, k^8\, \gamma^{1+k}+96\, k^9\, \gamma^{1+k}-64\, k\, \gamma^{2+k}-$$
$$280\, k^2\, \gamma^{2+k}+92\, k^3\, \gamma^{2+k}+1608\, k^4\, \gamma^{2+k}+1312\, k^5\, \gamma^{2+k}-1664\, k^6\, \gamma^{2+k}-2588\, k^7\, \gamma^{2+k}-1152\, k^8\, \gamma^{2+k}-$$
$$144\, k^9\, \gamma^{2+k}-144\, \gamma^{3+k}-528\, k\, \gamma^{3+k}-396\, k^2\, \gamma^{3+k}+636\, k^3\, \gamma^{3+k}+2272\, k^4\, \gamma^{3+k}+4932\, k^5\, \gamma^{3+k}+$$
$$6196\, k^6\, \gamma^{3+k}+3864\, k^7\, \gamma^{3+k}+1072\, k^8\, \gamma^{3+k}+96\, k^9\, \gamma^{3+k}+576\, \gamma^{4+k}+3408\, k\, \gamma^{4+k}+7848\, k^2\, \gamma^{4+k}+$$
$$8508\, k^3\, \gamma^{4+k}+3038\, k^4\, \gamma^{4+k}-2674\, k^5\, \gamma^{4+k}-3558\, k^6\, \gamma^{4+k}-1658\, k^7\, \gamma^{4+k}-344\, k^8\, \gamma^{4+k}-24\, k^9\, \gamma^{4+k}+$$
$$72\, \gamma^{3+2\,k}+384\, k\, \gamma^{3+2\,k}+78\, k^2\, \gamma^{3+2\,k}-2892\, k^3\, \gamma^{3+2\,k}-6390\, k^4\, \gamma^{3+2\,k}-5076\, k^5\, \gamma^{3+2\,k}-1296\, k^6\, \gamma^{3+2\,k}-$$
$$288\, \gamma^{4+2\,k}-1896\, k\, \gamma^{4+2\,k}-3996\, k^2\, \gamma^{4+2\,k}-2100\, k^3\, \gamma^{4+2\,k}+2988\, k^4\, \gamma^{4+2\,k}+3996\, k^5\, \gamma^{4+2\,k}+1296\, k^6\, \gamma^{4+2\,k}$$

In[58]:=
```
g7[1, k]
```

Out[58]=
$$0$$

In[59]:=
```
Factor[D[g7[γ, k], γ]]
```

Out[59]=
$$-\frac{1}{\gamma}2\, \left(2+k\right)$$
$$\left(-6\, k\, \gamma-k^2\, \gamma+16\, k^3\, \gamma-23\, k^4\, \gamma-24\, k^5\, \gamma+4\, k^6\, \gamma+14\, k^7\, \gamma-4\, k^8\, \gamma-32\, k\, \gamma^2-76\, k^2\, \gamma^2-26\, k^3\, \gamma^2-\right.$$
$$52\, k^4\, \gamma^2-116\, k^5\, \gamma^2-28\, k^6\, \gamma^2+30\, k^7\, \gamma^2+12\, k^8\, \gamma^2-54\, \gamma^3-81\, k\, \gamma^3+15\, k^2\, \gamma^3-33\, k^3\, \gamma^3-339\, k^4\, \gamma^3-$$
$$504\, k^5\, \gamma^3-330\, k^6\, \gamma^3-102\, k^7\, \gamma^3-12\, k^8\, \gamma^3+288\, \gamma^4+1368\, k\, \gamma^4+2836\, k^2\, \gamma^4+3372\, k^3\, \gamma^4+$$
$$2518\, k^4\, \gamma^4+1202\, k^5\, \gamma^4+354\, k^6\, \gamma^4+58\, k^7\, \gamma^4+4\, k^8\, \gamma^4+12\, k^2\, \gamma^k-44\, k^3\, \gamma^k-15\, k^4\, \gamma^k+119\, k^5\, \gamma^k+$$
$$19\, k^6\, \gamma^k-75\, k^7\, \gamma^k-4\, k^8\, \gamma^k+12\, k^9\, \gamma^k+6\, k\, \gamma^{1+k}+8\, k^2\, \gamma^{1+k}-86\, k^3\, \gamma^{1+k}-134\, k^4\, \gamma^{1+k}+244\, k^5\, \gamma^{1+k}+$$
$$454\, k^6\, \gamma^{1+k}+28\, k^7\, \gamma^{1+k}-184\, k^8\, \gamma^{1+k}-48\, k^9\, \gamma^{1+k}+32\, k\, \gamma^{2+k}+140\, k^2\, \gamma^{2+k}-46\, k^3\, \gamma^{2+k}-$$
$$804\, k^4\, \gamma^{2+k}-656\, k^5\, \gamma^{2+k}+832\, k^6\, \gamma^{2+k}+1294\, k^7\, \gamma^{2+k}+576\, k^8\, \gamma^{2+k}+72\, k^9\, \gamma^{2+k}+108\, \gamma^{3+k}+$$
$$378\, k\, \gamma^{3+k}+240\, k^2\, \gamma^{3+k}-498\, k^3\, \gamma^{3+k}-1614\, k^4\, \gamma^{3+k}-3460\, k^5\, \gamma^{3+k}-4150\, k^6\, \gamma^{3+k}-2372\, k^7\, \gamma^{3+k}-$$
$$584\, k^8\, \gamma^{3+k}-48\, k^9\, \gamma^{3+k}-576\, \gamma^{4+k}-3264\, k\, \gamma^{4+k}-7068\, k^2\, \gamma^{4+k}-6936\, k^3\, \gamma^{4+k}-1697\, k^4\, \gamma^{4+k}+$$
$$2763\, k^5\, \gamma^{4+k}+2845\, k^6\, \gamma^{4+k}+1125\, k^7\, \gamma^{4+k}+196\, k^8\, \gamma^{4+k}+12\, k^9\, \gamma^{4+k}-54\, \gamma^{3+2\,k}-297\, k\, \gamma^{3+2\,k}-$$
$$102\, k^2\, \gamma^{3+2\,k}+2181\, k^3\, \gamma^{3+2\,k}+5148\, k^4\, \gamma^{3+2\,k}+4428\, k^5\, \gamma^{3+2\,k}+1296\, k^6\, \gamma^{3+2\,k}+288\, \gamma^{4+2\,k}+$$
$$\left.1896\, k\, \gamma^{4+2\,k}+3996\, k^2\, \gamma^{4+2\,k}+2100\, k^3\, \gamma^{4+2\,k}-2988\, k^4\, \gamma^{4+2\,k}-3996\, k^5\, \gamma^{4+2\,k}-1296\, k^6\, \gamma^{4+2\,k}\right)$$

In[60]:=
```
g8[γ_, k_] = Expand[Factor[D[g7[γ, k], γ]] / (2 (2 + k))]
```

Out[60]=
$$6\,k + k^2 - 16\,k^3 + 23\,k^4 + 24\,k^5 - 4\,k^6 - 14\,k^7 + 4\,k^8 + 32\,k\,\gamma + 76\,k^2\,\gamma + 26\,k^3\,\gamma + 52\,k^4\,\gamma + 116\,k^5\,\gamma + 28\,k^6\,\gamma - 30\,k^7\,\gamma - 12\,k^8\,\gamma + 54\,\gamma^2 + 81\,k\,\gamma^2 - 15\,k^2\,\gamma^2 + 33\,k^3\,\gamma^2 + 339\,k^4\,\gamma^2 + 504\,k^5\,\gamma^2 + 330\,k^6\,\gamma^2 + 102\,k^7\,\gamma^2 + 12\,k^8\,\gamma^2 - 288\,\gamma^3 - 1368\,k\,\gamma^3 - 2836\,k^2\,\gamma^3 - 3372\,k^3\,\gamma^3 - 2518\,k^4\,\gamma^3 - 1202\,k^5\,\gamma^3 - 354\,k^6\,\gamma^3 - 58\,k^7\,\gamma^3 - 4\,k^8\,\gamma^3 - 12\,k^2\,\gamma^{-1+k} + 44\,k^3\,\gamma^{-1+k} + 15\,k^4\,\gamma^{-1+k} - 119\,k^5\,\gamma^{-1+k} - 19\,k^6\,\gamma^{-1+k} + 75\,k^7\,\gamma^{-1+k} + 4\,k^8\,\gamma^{-1+k} - 12\,k^9\,\gamma^{-1+k} - 6\,k\,\gamma^k - 8\,k^2\,\gamma^k + 86\,k^3\,\gamma^k + 134\,k^4\,\gamma^k - 244\,k^5\,\gamma^k - 454\,k^6\,\gamma^k - 28\,k^7\,\gamma^k + 184\,k^8\,\gamma^k + 48\,k^9\,\gamma^k - 32\,k\,\gamma^{1+k} - 140\,k^2\,\gamma^{1+k} + 46\,k^3\,\gamma^{1+k} + 804\,k^4\,\gamma^{1+k} + 656\,k^5\,\gamma^{1+k} - 832\,k^6\,\gamma^{1+k} - 1294\,k^7\,\gamma^{1+k} - 576\,k^8\,\gamma^{1+k} - 72\,k^9\,\gamma^{1+k} - 108\,\gamma^{2+k} - 378\,k\,\gamma^{2+k} - 240\,k^2\,\gamma^{2+k} + 498\,k^3\,\gamma^{2+k} + 1614\,k^4\,\gamma^{2+k} + 3460\,k^5\,\gamma^{2+k} + 4150\,k^6\,\gamma^{2+k} + 2372\,k^7\,\gamma^{2+k} + 584\,k^8\,\gamma^{2+k} + 48\,k^9\,\gamma^{2+k} + 576\,\gamma^{3+k} + 3264\,k\,\gamma^{3+k} + 7068\,k^2\,\gamma^{3+k} + 6936\,k^3\,\gamma^{3+k} + 1697\,k^4\,\gamma^{3+k} - 2763\,k^5\,\gamma^{3+k} - 2845\,k^6\,\gamma^{3+k} - 1125\,k^7\,\gamma^{3+k} - 196\,k^8\,\gamma^{3+k} - 12\,k^9\,\gamma^{3+k} + 54\,\gamma^{2+2\,k} + 297\,k\,\gamma^{2+2\,k} + 102\,k^2\,\gamma^{2+2\,k} - 2181\,k^3\,\gamma^{2+2\,k} - 5148\,k^4\,\gamma^{2+2\,k} - 4428\,k^5\,\gamma^{2+2\,k} - 1296\,k^6\,\gamma^{2+2\,k} - 288\,\gamma^{3+2\,k} - 1896\,k\,\gamma^{3+2\,k} - 3996\,k^2\,\gamma^{3+2\,k} - 2100\,k^3\,\gamma^{3+2\,k} + 2988\,k^4\,\gamma^{3+2\,k} + 3996\,k^5\,\gamma^{3+2\,k} + 1296\,k^6\,\gamma^{3+2\,k}$$

In[61]:=
```
g8[1, k]
```

Out[61]=
$$0$$

In[62]:=
```
Factor[D[g8[γ, k], γ]]
```

Out[62]=
$$-\frac{1}{\gamma^2}(1+k)\left(-32\,k\,\gamma^2 - 44\,k^2\,\gamma^2 + 18\,k^3\,\gamma^2 - 70\,k^4\,\gamma^2 - 46\,k^5\,\gamma^2 + 18\,k^6\,\gamma^2 + 12\,k^7\,\gamma^2 - 108\,\gamma^3 - 54\,k\,\gamma^3 + 84\,k^2\,\gamma^3 - 150\,k^3\,\gamma^3 - 528\,k^4\,\gamma^3 - 480\,k^5\,\gamma^3 - 180\,k^6\,\gamma^3 - 24\,k^7\,\gamma^3 + 864\,\gamma^4 + 3240\,k\,\gamma^4 + 5268\,k^2\,\gamma^4 + 4848\,k^3\,\gamma^4 + 2706\,k^4\,\gamma^4 + 900\,k^5\,\gamma^4 + 162\,k^6\,\gamma^4 + 12\,k^7\,\gamma^4 - 12\,k^2\,\gamma^k + 68\,k^3\,\gamma^k - 97\,k^4\,\gamma^k - 37\,k^5\,\gamma^k + 137\,k^6\,\gamma^k - 43\,k^7\,\gamma^k - 28\,k^8\,\gamma^k + 12\,k^9\,\gamma^k + 6\,k^2\,\gamma^{1+k} + 2\,k^3\,\gamma^{1+k} - 88\,k^4\,\gamma^{1+k} - 46\,k^5\,\gamma^{1+k} + 290\,k^6\,\gamma^{1+k} + 164\,k^7\,\gamma^{1+k} - 136\,k^8\,\gamma^{1+k} - 48\,k^9\,\gamma^{1+k} + 32\,k\,\gamma^{2+k} + 140\,k^2\,\gamma^{2+k} - 46\,k^3\,\gamma^{2+k} - 804\,k^4\,\gamma^{2+k} - 656\,k^5\,\gamma^{2+k} + 832\,k^6\,\gamma^{2+k} + 1294\,k^7\,\gamma^{2+k} + 576\,k^8\,\gamma^{2+k} + 72\,k^9\,\gamma^{2+k} + 216\,\gamma^{3+k} + 648\,k\,\gamma^{3+k} + 210\,k^2\,\gamma^{3+k} - 966\,k^3\,\gamma^{3+k} - 2760\,k^4\,\gamma^{3+k} - 5774\,k^5\,\gamma^{3+k} - 5986\,k^6\,\gamma^{3+k} - 2908\,k^7\,\gamma^{3+k} - 632\,k^8\,\gamma^{3+k} - 48\,k^9\,\gamma^{3+k} - 1728\,\gamma^{4+k} - 8640\,k\,\gamma^{4+k} - 15\,828\,k^2\,\gamma^{4+k} - 12\,048\,k^3\,\gamma^{4+k} + 21\,k^4\,\gamma^{4+k} + 6571\,k^5\,\gamma^{4+k} + 4727\,k^6\,\gamma^{4+k} + 1493\,k^7\,\gamma^{4+k} + 220\,k^8\,\gamma^{4+k} + 12\,k^9\,\gamma^{4+k} - 108\,\gamma^{3+2\,k} - 594\,k\,\gamma^{3+2\,k} - 204\,k^2\,\gamma^{3+2\,k} + 4362\,k^3\,\gamma^{3+2\,k} + 10\,296\,k^4\,\gamma^{3+2\,k} + 8856\,k^5\,\gamma^{3+2\,k} + 2592\,k^6\,\gamma^{3+2\,k} + 864\,\gamma^{4+2\,k} + 5400\,k\,\gamma^{4+2\,k} + 10\,380\,k^2\,\gamma^{4+2\,k} + 3912\,k^3\,\gamma^{4+2\,k} - 8676\,k^4\,\gamma^{4+2\,k} - 9288\,k^5\,\gamma^{4+2\,k} - 2592\,k^6\,\gamma^{4+2\,k}\right)$$

In[63]:=
```
g9[γ_, k_] = Expand[Factor[D[g8[γ, k], γ]] / (1 + k)]
```

Out[63]=
$$32\,k + 44\,k^2 - 18\,k^3 + 70\,k^4 + 46\,k^5 - 18\,k^6 - 12\,k^7 + 108\,\gamma + 54\,k\,\gamma - 84\,k^2\,\gamma + 150\,k^3\,\gamma + 528\,k^4\,\gamma + 480\,k^5\,\gamma + 180\,k^6\,\gamma + 24\,k^7\,\gamma - 864\,\gamma^2 - 3240\,k\,\gamma^2 - 5268\,k^2\,\gamma^2 - 4848\,k^3\,\gamma^2 - 2706\,k^4\,\gamma^2 - 900\,k^5\,\gamma^2 - 162\,k^6\,\gamma^2 - 12\,k^7\,\gamma^2 + 12\,k^2\,\gamma^{-2+k} - 68\,k^3\,\gamma^{-2+k} + 97\,k^4\,\gamma^{-2+k} + 37\,k^5\,\gamma^{-2+k} - 137\,k^6\,\gamma^{-2+k} + 43\,k^7\,\gamma^{-2+k} + 28\,k^8\,\gamma^{-2+k} - 12\,k^9\,\gamma^{-2+k} - 6\,k^2\,\gamma^{-1+k} - 2\,k^3\,\gamma^{-1+k} + 88\,k^4\,\gamma^{-1+k} + 46\,k^5\,\gamma^{-1+k} - 290\,k^6\,\gamma^{-1+k} - 164\,k^7\,\gamma^{-1+k} + 136\,k^8\,\gamma^{-1+k} + 48\,k^9\,\gamma^{-1+k} - 32\,k\,\gamma^k - 140\,k^2\,\gamma^k + 46\,k^3\,\gamma^k + 804\,k^4\,\gamma^k + 656\,k^5\,\gamma^k - 832\,k^6\,\gamma^k - 1294\,k^7\,\gamma^k - 576\,k^8\,\gamma^k - 72\,k^9\,\gamma^k - 216\,\gamma^{1+k} - 648\,k\,\gamma^{1+k} - 210\,k^2\,\gamma^{1+k} + 966\,k^3\,\gamma^{1+k} + 2760\,k^4\,\gamma^{1+k} + 5774\,k^5\,\gamma^{1+k} + 5986\,k^6\,\gamma^{1+k} + 2908\,k^7\,\gamma^{1+k} + 632\,k^8\,\gamma^{1+k} + 48\,k^9\,\gamma^{1+k} + 1728\,\gamma^{2+k} + 8640\,k\,\gamma^{2+k} + 15\,828\,k^2\,\gamma^{2+k} + 12\,048\,k^3\,\gamma^{2+k} - 21\,k^4\,\gamma^{2+k} - 6571\,k^5\,\gamma^{2+k} - 4727\,k^6\,\gamma^{2+k} - 1493\,k^7\,\gamma^{2+k} - 220\,k^8\,\gamma^{2+k} - 12\,k^9\,\gamma^{2+k} + 108\,\gamma^{1+2\,k} + 594\,k\,\gamma^{1+2\,k} + 204\,k^2\,\gamma^{1+2\,k} - 4362\,k^3\,\gamma^{1+2\,k} - 10\,296\,k^4\,\gamma^{1+2\,k} - 8856\,k^5\,\gamma^{1+2\,k} - 2592\,k^6\,\gamma^{1+2\,k} - 864\,\gamma^{2+2\,k} - 5400\,k\,\gamma^{2+2\,k} - 10\,380\,k^2\,\gamma^{2+2\,k} - 3912\,k^3\,\gamma^{2+2\,k} + 8676\,k^4\,\gamma^{2+2\,k} + 9288\,k^5\,\gamma^{2+2\,k} + 2592\,k^6\,\gamma^{2+2\,k}$$

In[64]:=
```
g9[1, k]
```

Out[64]=
$$0$$

In[65]:=
```
Factor[D[g9[γ, k], γ]]
```

Out[65]=
$$-\frac{1}{\gamma^3}\left(-108\,\gamma^3-54\,k\,\gamma^3+84\,k^2\,\gamma^3-150\,k^3\,\gamma^3-528\,k^4\,\gamma^3-480\,k^5\,\gamma^3-180\,k^6\,\gamma^3-24\,k^7\,\gamma^3+1728\,\gamma^4+6480\,k\,\gamma^4+10\,536\,k^2\,\gamma^4+9696\,k^3\,\gamma^4+5412\,k^4\,\gamma^4+1800\,k^5\,\gamma^4+324\,k^6\,\gamma^4+24\,k^7\,\gamma^4+24\,k^2\,\gamma^k-148\,k^3\,\gamma^k+262\,k^4\,\gamma^k-23\,k^5\,\gamma^k-311\,k^6\,\gamma^k+223\,k^7\,\gamma^k+13\,k^8\,\gamma^k-52\,k^9\,\gamma^k+12\,k^{10}\,\gamma^k-6\,k^2\,\gamma^{1+k}+4\,k^3\,\gamma^{1+k}+90\,k^4\,\gamma^{1+k}-42\,k^5\,\gamma^{1+k}-336\,k^6\,\gamma^{1+k}+126\,k^7\,\gamma^{1+k}+300\,k^8\,\gamma^{1+k}-88\,k^9\,\gamma^{1+k}-48\,k^{10}\,\gamma^{1+k}+32\,k^2\,\gamma^{2+k}+140\,k^3\,\gamma^{2+k}-46\,k^4\,\gamma^{2+k}-804\,k^5\,\gamma^{2+k}-656\,k^6\,\gamma^{2+k}+832\,k^7\,\gamma^{2+k}+1294\,k^8\,\gamma^{2+k}+576\,k^9\,\gamma^{2+k}+72\,k^{10}\,\gamma^{2+k}+216\,\gamma^{3+k}+864\,k\,\gamma^{3+k}+858\,k^2\,\gamma^{3+k}-756\,k^3\,\gamma^{3+k}-3726\,k^4\,\gamma^{3+k}-8534\,k^5\,\gamma^{3+k}-11\,760\,k^6\,\gamma^{3+k}-8894\,k^7\,\gamma^{3+k}-3540\,k^8\,\gamma^{3+k}-680\,k^9\,\gamma^{3+k}-48\,k^{10}\,\gamma^{3+k}-3456\,\gamma^{4+k}-19\,008\,k\,\gamma^{4+k}-40\,296\,k^2\,\gamma^{4+k}-39\,924\,k^3\,\gamma^{4+k}-12\,006\,k^4\,\gamma^{4+k}+13\,163\,k^5\,\gamma^{4+k}+16\,025\,k^6\,\gamma^{4+k}+7713\,k^7\,\gamma^{4+k}+1933\,k^8\,\gamma^{4+k}+244\,k^9\,\gamma^{4+k}+12\,k^{10}\,\gamma^{4+k}-108\,\gamma^{3+2k}-810\,k\,\gamma^{3+2k}-1392\,k^2\,\gamma^{3+2k}+3954\,k^3\,\gamma^{3+2k}+19\,020\,k^4\,\gamma^{3+2k}+29\,448\,k^5\,\gamma^{3+2k}+20\,304\,k^6\,\gamma^{3+2k}+5184\,k^7\,\gamma^{3+2k}+1728\,\gamma^{4+2k}+12\,528\,k\,\gamma^{4+2k}+31\,560\,k^2\,\gamma^{4+2k}+28\,584\,k^3\,\gamma^{4+2k}-9528\,k^4\,\gamma^{4+2k}-35\,928\,k^5\,\gamma^{4+2k}-23\,760\,k^6\,\gamma^{4+2k}-5184\,k^7\,\gamma^{4+2k}\right)$$

In[69]:=
```
g10[γ_, k_] = Expand[Factor[D[g9[γ, k], γ]]]
```

Out[69]=
$$108+54\,k-84\,k^2+150\,k^3+528\,k^4+480\,k^5+180\,k^6+24\,k^7-1728\,\gamma-6480\,k\,\gamma-10\,536\,k^2\,\gamma-9696\,k^3\,\gamma-5412\,k^4\,\gamma-1800\,k^5\,\gamma-324\,k^6\,\gamma-24\,k^7\,\gamma-24\,k^2\,\gamma^{-3+k}+148\,k^3\,\gamma^{-3+k}-262\,k^4\,\gamma^{-3+k}+23\,k^5\,\gamma^{-3+k}+311\,k^6\,\gamma^{-3+k}-223\,k^7\,\gamma^{-3+k}-13\,k^8\,\gamma^{-3+k}+52\,k^9\,\gamma^{-3+k}-12\,k^{10}\,\gamma^{-3+k}+6\,k^2\,\gamma^{-2+k}-4\,k^3\,\gamma^{-2+k}-90\,k^4\,\gamma^{-2+k}+42\,k^5\,\gamma^{-2+k}+336\,k^6\,\gamma^{-2+k}-126\,k^7\,\gamma^{-2+k}-300\,k^8\,\gamma^{-2+k}+88\,k^9\,\gamma^{-2+k}+48\,k^{10}\,\gamma^{-2+k}-32\,k^2\,\gamma^{-1+k}-140\,k^3\,\gamma^{-1+k}+46\,k^4\,\gamma^{-1+k}+804\,k^5\,\gamma^{-1+k}+656\,k^6\,\gamma^{-1+k}-832\,k^7\,\gamma^{-1+k}-1294\,k^8\,\gamma^{-1+k}-576\,k^9\,\gamma^{-1+k}-72\,k^{10}\,\gamma^{-1+k}-216\,\gamma^k-864\,k\,\gamma^k-858\,k^2\,\gamma^k+756\,k^3\,\gamma^k+3726\,k^4\,\gamma^k+8534\,k^5\,\gamma^k+11\,760\,k^6\,\gamma^k+8894\,k^7\,\gamma^k+3540\,k^8\,\gamma^k+680\,k^9\,\gamma^k+48\,k^{10}\,\gamma^k+108\,\gamma^{2k}+810\,k\,\gamma^{2k}+1392\,k^2\,\gamma^{2k}-3954\,k^3\,\gamma^{2k}-19\,020\,k^4\,\gamma^{2k}-29\,448\,k^5\,\gamma^{2k}-20\,304\,k^6\,\gamma^{2k}-5184\,k^7\,\gamma^{2k}+3456\,\gamma^{1+k}+19\,008\,k\,\gamma^{1+k}+40\,296\,k^2\,\gamma^{1+k}+39\,924\,k^3\,\gamma^{1+k}+12\,006\,k^4\,\gamma^{1+k}-13\,163\,k^5\,\gamma^{1+k}-16\,025\,k^6\,\gamma^{1+k}-7713\,k^7\,\gamma^{1+k}-1933\,k^8\,\gamma^{1+k}-244\,k^9\,\gamma^{1+k}-12\,k^{10}\,\gamma^{1+k}-1728\,\gamma^{1+2k}-12\,528\,k\,\gamma^{1+2k}-31\,560\,k^2\,\gamma^{1+2k}-28\,584\,k^3\,\gamma^{1+2k}+9528\,k^4\,\gamma^{1+2k}+35\,928\,k^5\,\gamma^{1+2k}+23\,760\,k^6\,\gamma^{1+2k}+5184\,k^7\,\gamma^{1+2k}$$

In[75]:=
```
Factor[g10[1, k]]
```

Out[75]=
$$350\,(-1+k)\,k^2\,(1+k)\,(2+k)^2$$

```
(*Note here that g10[1,k]≥0 instead of always 0.
   The same will be true for g11[1,k]-g16[1,k]*)
```

In[72]:=
```
Factor[D[g10[γ, k], γ]]
```

Out[72]=
$$-\frac{1}{\gamma^4}\left(1728\,\gamma^4+6480\,k\,\gamma^4+10\,536\,k^2\,\gamma^4+9696\,k^3\,\gamma^4+5412\,k^4\,\gamma^4+1800\,k^5\,\gamma^4+324\,k^6\,\gamma^4+24\,k^7\,\gamma^4-72\,k^2\,\gamma^k+468\,k^3\,\gamma^k-934\,k^4\,\gamma^k+331\,k^5\,\gamma^k+910\,k^6\,\gamma^k-980\,k^7\,\gamma^k+184\,k^8\,\gamma^k+169\,k^9\,\gamma^k-88\,k^{10}\,\gamma^k+12\,k^{11}\,\gamma^k+12\,k^2\,\gamma^{1+k}-14\,k^3\,\gamma^{1+k}-176\,k^4\,\gamma^{1+k}+174\,k^5\,\gamma^{1+k}+630\,k^6\,\gamma^{1+k}-588\,k^7\,\gamma^{1+k}-474\,k^8\,\gamma^{1+k}+476\,k^9\,\gamma^{1+k}+8\,k^{10}\,\gamma^{1+k}-48\,k^{11}\,\gamma^{1+k}-32\,k^2\,\gamma^{2+k}-108\,k^3\,\gamma^{2+k}+186\,k^4\,\gamma^{2+k}+758\,k^5\,\gamma^{2+k}-148\,k^6\,\gamma^{2+k}-1488\,k^7\,\gamma^{2+k}-462\,k^8\,\gamma^{2+k}+718\,k^9\,\gamma^{2+k}+504\,k^{10}\,\gamma^{2+k}+72\,k^{11}\,\gamma^{2+k}+216\,k\,\gamma^{3+k}+864\,k^2\,\gamma^{3+k}+858\,k^3\,\gamma^{3+k}-756\,k^4\,\gamma^{3+k}-3726\,k^5\,\gamma^{3+k}-8534\,k^6\,\gamma^{3+k}-11\,760\,k^7\,\gamma^{3+k}-8894\,k^8\,\gamma^{3+k}-3540\,k^9\,\gamma^{3+k}-680\,k^{10}\,\gamma^{3+k}-48\,k^{11}\,\gamma^{3+k}-3456\,\gamma^{4+k}-22\,464\,k\,\gamma^{4+k}-59\,304\,k^2\,\gamma^{4+k}-80\,220\,k^3\,\gamma^{4+k}-51\,930\,k^4\,\gamma^{4+k}+1157\,k^5\,\gamma^{4+k}+29\,188\,k^6\,\gamma^{4+k}+23\,738\,k^7\,\gamma^{4+k}+9646\,k^8\,\gamma^{4+k}+2177\,k^9\,\gamma^{4+k}+256\,k^{10}\,\gamma^{4+k}+12\,k^{11}\,\gamma^{4+k}-216\,k\,\gamma^{3+2k}-1620\,k^2\,\gamma^{3+2k}-2784\,k^3\,\gamma^{3+2k}+7908\,k^4\,\gamma^{3+2k}+38\,040\,k^5\,\gamma^{3+2k}+58\,896\,k^6\,\gamma^{3+2k}+40\,608\,k^7\,\gamma^{3+2k}+10\,368\,k^8\,\gamma^{3+2k}+1728\,\gamma^{4+2k}+15\,984\,k\,\gamma^{4+2k}+56\,616\,k^2\,\gamma^{4+2k}+91\,704\,k^3\,\gamma^{4+2k}+47\,640\,k^4\,\gamma^{4+2k}-54\,984\,k^5\,\gamma^{4+2k}-95\,616\,k^6\,\gamma^{4+2k}-52\,704\,k^7\,\gamma^{4+2k}-10\,368\,k^8\,\gamma^{4+2k}\right)$$

In[73]:=
```
g11[γ_, k_] = Expand[Factor[D[g10[γ, k], γ]]]
```

Out[73]=
$$-1728-6480\,k-10\,536\,k^2-9696\,k^3-5412\,k^4-1800\,k^5-324\,k^6-24\,k^7+72\,k^2\,\gamma^{-4+k}-468\,k^3\,\gamma^{-4+k}+934\,k^4\,\gamma^{-4+k}-331\,k^5\,\gamma^{-4+k}-910\,k^6\,\gamma^{-4+k}+980\,k^7\,\gamma^{-4+k}-184\,k^8\,\gamma^{-4+k}-169\,k^9\,\gamma^{-4+k}+88\,k^{10}\,\gamma^{-4+k}-12\,k^{11}\,\gamma^{-4+k}-12\,k^2\,\gamma^{-3+k}+14\,k^3\,\gamma^{-3+k}+176\,k^4\,\gamma^{-3+k}-174\,k^5\,\gamma^{-3+k}-630\,k^6\,\gamma^{-3+k}+588\,k^7\,\gamma^{-3+k}+474\,k^8\,\gamma^{-3+k}-476\,k^9\,\gamma^{-3+k}-8\,k^{10}\,\gamma^{-3+k}+48\,k^{11}\,\gamma^{-3+k}+32\,k^2\,\gamma^{-2+k}+108\,k^3\,\gamma^{-2+k}-186\,k^4\,\gamma^{-2+k}-758\,k^5\,\gamma^{-2+k}+148\,k^6\,\gamma^{-2+k}+1488\,k^7\,\gamma^{-2+k}+462\,k^8\,\gamma^{-2+k}-718\,k^9\,\gamma^{-2+k}-504\,k^{10}\,\gamma^{-2+k}-72\,k^{11}\,\gamma^{-2+k}-216\,k\,\gamma^{-1+k}-864\,k^2\,\gamma^{-1+k}-858\,k^3\,\gamma^{-1+k}+756\,k^4\,\gamma^{-1+k}+3726\,k^5\,\gamma^{-1+k}+8534\,k^6\,\gamma^{-1+k}+11\,760\,k^7\,\gamma^{-1+k}+8894\,k^8\,\gamma^{-1+k}+3540\,k^9\,\gamma^{-1+k}+680\,k^{10}\,\gamma^{-1+k}+48\,k^{11}\,\gamma^{-1+k}+3456\,\gamma^k+22\,464\,k\,\gamma^k+59\,304\,k^2\,\gamma^k+80\,220\,k^3\,\gamma^k+51\,930\,k^4\,\gamma^k-1157\,k^5\,\gamma^k-29\,188\,k^6\,\gamma^k-23\,738\,k^7\,\gamma^k-9646\,k^8\,\gamma^k-2177\,k^9\,\gamma^k-256\,k^{10}\,\gamma^k-12\,k^{11}\,\gamma^k-1728\,\gamma^{2\,k}-15\,984\,k\,\gamma^{2\,k}-56\,616\,k^2\,\gamma^{2\,k}-91\,704\,k^3\,\gamma^{2\,k}-47\,640\,k^4\,\gamma^{2\,k}+54\,984\,k^5\,\gamma^{2\,k}+95\,616\,k^6\,\gamma^{2\,k}+52\,704\,k^7\,\gamma^{2\,k}+10\,368\,k^8\,\gamma^{2\,k}+216\,k\,\gamma^{-1+2\,k}+1620\,k^2\,\gamma^{-1+2\,k}+2784\,k^3\,\gamma^{-1+2\,k}-7908\,k^4\,\gamma^{-1+2\,k}-38\,040\,k^5\,\gamma^{-1+2\,k}-58\,896\,k^6\,\gamma^{-1+2\,k}-40\,608\,k^7\,\gamma^{-1+2\,k}-10\,368\,k^8\,\gamma^{-1+2\,k}$$

In[76]:=
```
Factor[g11[1, k]]
```

Out[76]=
$$350\,(-1+k)\,k^2\,(1+k)\,(2+k)^2\,(5+9\,k)$$

In[77]:=
```
Factor[D[g11[γ, k], γ]]
```

Out[77]=
$$k\,\gamma^{-5+k}\,(-288\,k+1944\,k^2-4204\,k^3+2258\,k^4+3309\,k^5-4830\,k^6+1716\,k^7+492\,k^8-521\,k^9+136\,k^{10}-12\,k^{11}+36\,k\,\gamma-54\,k^2\,\gamma-514\,k^3\,\gamma+698\,k^4\,\gamma+1716\,k^5\,\gamma-2394\,k^6\,\gamma-834\,k^7\,\gamma+1902\,k^8\,\gamma-452\,k^9\,\gamma-152\,k^{10}\,\gamma+48\,k^{11}\,\gamma-64\,k\,\gamma^2-184\,k^2\,\gamma^2+480\,k^3\,\gamma^2+1330\,k^4\,\gamma^2-1054\,k^5\,\gamma^2-2828\,k^6\,\gamma^2+564\,k^7\,\gamma^2+1898\,k^8\,\gamma^2+290\,k^9\,\gamma^2-360\,k^{10}\,\gamma^2-72\,k^{11}\,\gamma^2+216\,\gamma^3+648\,k\,\gamma^3-6\,k^2\,\gamma^3-1614\,k^3\,\gamma^3-2970\,k^4\,\gamma^3-4808\,k^5\,\gamma^3-3226\,k^6\,\gamma^3+2866\,k^7\,\gamma^3+5354\,k^8\,\gamma^3+2860\,k^9\,\gamma^3+632\,k^{10}\,\gamma^3+48\,k^{11}\,\gamma^3+3456\,\gamma^4+22\,464\,k\,\gamma^4+59\,304\,k^2\,\gamma^4+80\,220\,k^3\,\gamma^4+51\,930\,k^4\,\gamma^4-1157\,k^5\,\gamma^4-29\,188\,k^6\,\gamma^4-23\,738\,k^7\,\gamma^4-9646\,k^8\,\gamma^4-2177\,k^9\,\gamma^4-256\,k^{10}\,\gamma^4-12\,k^{11}\,\gamma^4-216\,\gamma^{3+k}-1188\,k\,\gamma^{3+k}+456\,k^2\,\gamma^{3+k}+13\,476\,k^3\,\gamma^{3+k}+22\,224\,k^4\,\gamma^{3+k}-17\,184\,k^5\,\gamma^{3+k}-77\,184\,k^6\,\gamma^{3+k}-70\,848\,k^7\,\gamma^{3+k}-20\,736\,k^8\,\gamma^{3+k}-3456\,\gamma^{4+k}-31\,968\,k\,\gamma^{4+k}-113\,232\,k^2\,\gamma^{4+k}-183\,408\,k^3\,\gamma^{4+k}-95\,280\,k^4\,\gamma^{4+k}+109\,968\,k^5\,\gamma^{4+k}+191\,232\,k^6\,\gamma^{4+k}+105\,408\,k^7\,\gamma^{4+k}+20\,736\,k^8\,\gamma^{4+k})$$

In[78]:=
```
g12[γ_, k_] = Expand[Factor[D[g11[γ, k], γ]] / (k γ^(-5+k))]
```

Out[78]=
$$-288\,k+1944\,k^2-4204\,k^3+2258\,k^4+3309\,k^5-4830\,k^6+1716\,k^7+492\,k^8-521\,k^9+136\,k^{10}-12\,k^{11}+36\,k\,\gamma-54\,k^2\,\gamma-514\,k^3\,\gamma+698\,k^4\,\gamma+1716\,k^5\,\gamma-2394\,k^6\,\gamma-834\,k^7\,\gamma+1902\,k^8\,\gamma-452\,k^9\,\gamma-152\,k^{10}\,\gamma+48\,k^{11}\,\gamma-64\,k\,\gamma^2-184\,k^2\,\gamma^2+480\,k^3\,\gamma^2+1330\,k^4\,\gamma^2-1054\,k^5\,\gamma^2-2828\,k^6\,\gamma^2+564\,k^7\,\gamma^2+1898\,k^8\,\gamma^2+290\,k^9\,\gamma^2-360\,k^{10}\,\gamma^2-72\,k^{11}\,\gamma^2+216\,\gamma^3+648\,k\,\gamma^3-6\,k^2\,\gamma^3-1614\,k^3\,\gamma^3-2970\,k^4\,\gamma^3-4808\,k^5\,\gamma^3-3226\,k^6\,\gamma^3+2866\,k^7\,\gamma^3+5354\,k^8\,\gamma^3+2860\,k^9\,\gamma^3+632\,k^{10}\,\gamma^3+48\,k^{11}\,\gamma^3+3456\,\gamma^4+22\,464\,k\,\gamma^4+59\,304\,k^2\,\gamma^4+80\,220\,k^3\,\gamma^4+51\,930\,k^4\,\gamma^4-1157\,k^5\,\gamma^4-29\,188\,k^6\,\gamma^4-23\,738\,k^7\,\gamma^4-9646\,k^8\,\gamma^4-2177\,k^9\,\gamma^4-256\,k^{10}\,\gamma^4-12\,k^{11}\,\gamma^4-216\,\gamma^{3+k}-1188\,k\,\gamma^{3+k}+456\,k^2\,\gamma^{3+k}+13\,476\,k^3\,\gamma^{3+k}+22\,224\,k^4\,\gamma^{3+k}-17\,184\,k^5\,\gamma^{3+k}-77\,184\,k^6\,\gamma^{3+k}-70\,848\,k^7\,\gamma^{3+k}-20\,736\,k^8\,\gamma^{3+k}-3456\,\gamma^{4+k}-31\,968\,k\,\gamma^{4+k}-113\,232\,k^2\,\gamma^{4+k}-183\,408\,k^3\,\gamma^{4+k}-95\,280\,k^4\,\gamma^{4+k}+109\,968\,k^5\,\gamma^{4+k}+191\,232\,k^6\,\gamma^{4+k}+105\,408\,k^7\,\gamma^{4+k}+20\,736\,k^8\,\gamma^{4+k}$$

In[79]:=
```
Factor[g12[1, k]]
```

Out[79]=
$$14\,(-1+k)\,k\,(1+k)\,(2+k)\,(370+1664\,k+2951\,k^2+1081\,k^3)$$

In[80]:=
```
Factor[D[g12[γ, k], γ]]
```

Out[80]=
$$-2\,(1+2k)\,(-18k+63k^2+131k^3-611k^4+364k^5+469k^6-521k^7+91k^8+44k^9-12k^{10}+64k\gamma+56k^2\gamma-592k^3\gamma-146k^4\gamma+1346k^5\gamma+136k^6\gamma-836k^7\gamma-226k^8\gamma+162k^9\gamma+36k^{10}\gamma-324\gamma^2-324k\gamma^2+657k^2\gamma^2+1107k^3\gamma^2+2241k^4\gamma^2+2730k^5\gamma^2-621k^6\gamma^2-3057k^7\gamma^2-1917k^8\gamma^2-456k^9\gamma^2-36k^{10}\gamma^2-6912\gamma^3-31104k\gamma^3-56400k^2\gamma^3-47640k^3\gamma^3-8580k^4\gamma^3+19474k^5\gamma^3+19428k^6\gamma^3+8620k^7\gamma^3+2052k^8\gamma^3+250k^9\gamma^3+12k^{10}\gamma^3+324\gamma^{2+k}+1242k\gamma^{2+k}-2574k^2\gamma^{2+k}-15294k^3\gamma^{2+k}-9486k^4\gamma^{2+k}+33636k^5\gamma^{2+k}+57096k^6\gamma^{2+k}+30672k^7\gamma^{2+k}+5184k^8\gamma^{2+k}+6912\gamma^{3+k}+51840k\gamma^{3+k}+138768k^2\gamma^{3+k}+145896k^3\gamma^{3+k}-9528k^4\gamma^{3+k}-153240k^5\gamma^{3+k}-130968k^6\gamma^{3+k}-44496k^7\gamma^{3+k}-5184k^8\gamma^{3+k})$$

In[81]:=
```
g13[γ_, k_] = Expand[Factor[D[g12[γ, k], γ]] / (2 (1 + 2 k) )]
```

Out[81]=
$$18k-63k^2-131k^3+611k^4-364k^5-469k^6+521k^7-91k^8-44k^9+12k^{10}-64k\gamma-56k^2\gamma+592k^3\gamma+146k^4\gamma-1346k^5\gamma-136k^6\gamma+836k^7\gamma+226k^8\gamma-162k^9\gamma-36k^{10}\gamma+324\gamma^2+324k\gamma^2-657k^2\gamma^2-1107k^3\gamma^2-2241k^4\gamma^2-2730k^5\gamma^2+621k^6\gamma^2+3057k^7\gamma^2+1917k^8\gamma^2+456k^9\gamma^2+36k^{10}\gamma^2+6912\gamma^3+31104k\gamma^3+56400k^2\gamma^3+47640k^3\gamma^3+8580k^4\gamma^3-19474k^5\gamma^3-19428k^6\gamma^3-8620k^7\gamma^3-2052k^8\gamma^3-250k^9\gamma^3-12k^{10}\gamma^3-324\gamma^{2+k}-1242k\gamma^{2+k}+2574k^2\gamma^{2+k}+15294k^3\gamma^{2+k}+9486k^4\gamma^{2+k}-33636k^5\gamma^{2+k}-57096k^6\gamma^{2+k}-30672k^7\gamma^{2+k}-5184k^8\gamma^{2+k}-6912\gamma^{3+k}-51840k\gamma^{3+k}-138768k^2\gamma^{3+k}-145896k^3\gamma^{3+k}+9528k^4\gamma^{3+k}+153240k^5\gamma^{3+k}+130968k^6\gamma^{3+k}+44496k^7\gamma^{3+k}+5184k^8\gamma^{3+k}$$

In[82]:=
```
Factor[g13[1, k]]
```

Out[82]=
$$14\,(-1+k)\,k\,(1+k)\,(2+k)\,(775+2490k+2516k^2+687k^3)$$

In[83]:=
```
Factor[D[g13[γ, k], γ]]
```

Out[83]=
$$-2\,(32k+28k^2-296k^3-73k^4+673k^5+68k^6-418k^7-113k^8+81k^9+18k^{10}-324\gamma-324k\gamma+657k^2\gamma+1107k^3\gamma+2241k^4\gamma+2730k^5\gamma-621k^6\gamma-3057k^7\gamma-1917k^8\gamma-456k^9\gamma-36k^{10}\gamma-10368\gamma^2-46656k\gamma^2-84600k^2\gamma^2-71460k^3\gamma^2-12870k^4\gamma^2+29211k^5\gamma^2+29142k^6\gamma^2+12930k^7\gamma^2+3078k^8\gamma^2+375k^9\gamma^2+18k^{10}\gamma^2+324\gamma^{1+k}+1404k\gamma^{1+k}-1953k^2\gamma^{1+k}-16581k^3\gamma^{1+k}-17133k^4\gamma^{1+k}+28893k^5\gamma^{1+k}+73914k^6\gamma^{1+k}+59220k^7\gamma^{1+k}+20520k^8\gamma^{1+k}+2592k^9\gamma^{1+k}+10368\gamma^{2+k}+81216k\gamma^{2+k}+234072k^2\gamma^{2+k}+288228k^3\gamma^{2+k}+58656k^4\gamma^{2+k}-234624k^5\gamma^{2+k}-273072k^6\gamma^{2+k}-132228k^7\gamma^{2+k}-30024k^8\gamma^{2+k}-2592k^9\gamma^{2+k})$$

In[84]:=
```
g14[γ_, k_] = Expand[Factor[D[g13[γ, k], γ]] / (2 )]
```

Out[84]=
$$-32k-28k^2+296k^3+73k^4-673k^5-68k^6+418k^7+113k^8-81k^9-18k^{10}+324\gamma+324k\gamma-657k^2\gamma-1107k^3\gamma-2241k^4\gamma-2730k^5\gamma+621k^6\gamma+3057k^7\gamma+1917k^8\gamma+456k^9\gamma+36k^{10}\gamma+10368\gamma^2+46656k\gamma^2+84600k^2\gamma^2+71460k^3\gamma^2+12870k^4\gamma^2-29211k^5\gamma^2-29142k^6\gamma^2-12930k^7\gamma^2-3078k^8\gamma^2-375k^9\gamma^2-18k^{10}\gamma^2-324\gamma^{1+k}-1404k\gamma^{1+k}+1953k^2\gamma^{1+k}+16581k^3\gamma^{1+k}+17133k^4\gamma^{1+k}-28893k^5\gamma^{1+k}-73914k^6\gamma^{1+k}-59220k^7\gamma^{1+k}-20520k^8\gamma^{1+k}-2592k^9\gamma^{1+k}-10368\gamma^{2+k}-81216k\gamma^{2+k}-234072k^2\gamma^{2+k}-288228k^3\gamma^{2+k}-58656k^4\gamma^{2+k}+234624k^5\gamma^{2+k}+273072k^6\gamma^{2+k}+132228k^7\gamma^{2+k}+30024k^8\gamma^{2+k}+2592k^9\gamma^{2+k}$$

In[85]:=
```
Factor[g14[1, k]]
```

Out[85]=
$$7\,(-1+k)\,k\,(1+k)\,(2+k)\,(2548+9312k+12249k^2+6663k^3+1208k^4)$$

In[86]:=
```
Factor[D[g14[γ, k], γ]]
```

Out[86]=
$$3\,(1+k)\,(2+k)\,(3+k)\,(3+2k)\,\left(6-9k-3k^2+2k^3-33k^4+31k^5+6k^6+384\gamma+768k\gamma+360k^2\gamma-280k^3\gamma-306k^4\gamma-80k^5\gamma-6k^6\gamma-6\gamma^k-17k\gamma^k+66k^2\gamma^k+221k^3\gamma^k-60k^4\gamma^k-612k^5\gamma^k-432k^6\gamma^k-384\gamma^{1+k}-2240k\gamma^{1+k}-3720k^2\gamma^{1+k}-412k^3\gamma^{1+k}+3696k^4\gamma^{1+k}+2628k^5\gamma^{1+k}+432k^6\gamma^{1+k}\right)$$

In[87]:=
```
g15[γ_, k_] = Expand[Factor[D[g14[γ, k], γ]] / (3 (1 + k) (2 + k) (3 + k) (3 + 2 k))]
```

Out[87]=
$$6-9k-3k^2+2k^3-33k^4+31k^5+6k^6+384\gamma+768k\gamma+360k^2\gamma-280k^3\gamma-306k^4\gamma-80k^5\gamma-6k^6\gamma-6\gamma^k-17k\gamma^k+66k^2\gamma^k+221k^3\gamma^k-60k^4\gamma^k-612k^5\gamma^k-432k^6\gamma^k-384\gamma^{1+k}-2240k\gamma^{1+k}-3720k^2\gamma^{1+k}-412k^3\gamma^{1+k}+3696k^4\gamma^{1+k}+2628k^5\gamma^{1+k}+432k^6\gamma^{1+k}$$

In[88]:=
```
Factor[g15[1, k]]
```

Out[88]=
$$7\,(-1+k)\,k\,(1+k)\,(214+471k+281k^2)$$

In[90]:=
```
g16[γ_, k_] = Expand[Factor[D[g15[γ, k], γ]]]
```

Out[90]=
$$384+768k+360k^2-280k^3-306k^4-80k^5-6k^6-6k\gamma^{-1+k}-17k^2\gamma^{-1+k}+66k^3\gamma^{-1+k}+221k^4\gamma^{-1+k}-60k^5\gamma^{-1+k}-612k^6\gamma^{-1+k}-432k^7\gamma^{-1+k}-384\gamma^k-2624k\gamma^k-5960k^2\gamma^k-4132k^3\gamma^k+3284k^4\gamma^k+6324k^5\gamma^k+3060k^6\gamma^k+432k^7\gamma^k$$

In[91]:=
```
Factor[g16[1, k]]
```

Out[91]=
$$(-1+k)\,k\,(1862+7479k+11825k^2+8626k^3+2442k^4)$$

In[92]:=
```
Factor[D[g16[γ, k], γ]]
```

Out[92]=
$$(-1+k)\,k\,(1+3k)\,(2+3k)\,(3+4k)\,\gamma^{-2+k}\,(-1+3k+4k^2-12k^3+64\gamma+128k\gamma+76k^2\gamma+12k^3\gamma)$$

```
(*After 17 times...*)
```

In[93]:=
```
g17[γ_, k_] = Expand[Factor[D[g16[γ, k], γ]] / ((-1 + k) k (1 + 3 k) (2 + 3 k) (3 + 4 k) γ^(-2+k))]
```

Out[93]=
$$-1+3k+4k^2-12k^3+64\gamma+128k\gamma+76k^2\gamma+12k^3\gamma$$

```
(*Note that, since γ≥1,
    g17[γ,k] = -1+3 k+4 k^2-12 k^3+64 γ+128 k γ+76 k^2 γ+12 k^3 γ
             ≥ -1+3 k+4 k^2+64 γ+128 k γ+76 k^2 γ ≥ 0.
   This shows that g[γ,k]≥0 for k≥5.*)

(*We now check g[γ,k] for k≤4*)
```

In[95]:=
```
Factor[g[γ, 4]]
```

Out[95]=
$$4500\,(-1+\gamma)^{12}\,\gamma^4\,(1+8\gamma+35\gamma^2+110\gamma^3+212\gamma^4+268\gamma^5+212\gamma^6+110\gamma^7+35\gamma^8+8\gamma^9+\gamma^{10})$$

In[97]:=
```
Factor[g[γ, 3]]
```

Out[97]=
$$512\,(-1+\gamma)^{12}\,\gamma^3\,(2+15\gamma+60\gamma^2+96\gamma^3+60\gamma^4+15\gamma^5+2\gamma^6)$$

In[98]:=
```
Factor[g[γ, 2]]
```

Out[98]=
$$108\,(-1+\gamma)^{12}\,\gamma^2\,(1+6\gamma+\gamma^2)$$

In[99]:=
```
Factor[g[γ, 1]]
```

Out[99]=
$$0$$

```
(*Therefore, g[ɣ,k]≥ 0 for ɣ≥1 and k=1,2,3,4.*)

(*This completes the proof.*)
```